Example 6: A disc wheel starts to rotate from rest with constant angular acceleration. At time $t = 20$ s its frequency equals 200 revolutions per minute.

Determine the angular acceleration of the motion. How many revolutions are performed during this time?

Solution: The angular velocity of the wheel equals $\omega = 2\pi n$, where $n = \frac{200}{60}$ represents the number of revolutions per second.

So, the angular velocity at time $t = 20$ s equals

$$\omega = 2\pi n = \frac{2\pi.200}{60} \doteq 21\ s^{-1}.$$

Since the motion starts from rest the angular acceleration will be equal to

$$\varepsilon = \frac{\omega}{t} \doteq \frac{21}{20} = 1{,}05\ s^{-2}.$$

The angle subtended during time $t = 20$ s equals

$$\varphi(t) = \int_0^t \omega(t)\,dt = \int_0^t \varepsilon t\,dt = \frac{1}{2}\varepsilon t^2 = \frac{1}{2}\,1{,}05.20^2 = 210.$$

So, the number of revolutions during time $t = 20$ s equals

$$N = \frac{210}{2\pi} \doteq 33{,}4.$$

2. DYNAMICS

2-1 The First Law of Motion

In Chapter 1 we have discussed how motion is described in terms of velocity and acceleration. In this Chapter we want to deal with the question of why objects move as they do, what causes a body to accelerate or decelerate. We can answer that a force is required and therefore we investigate the connection between force and motion. A force has direction as well as magnitude and is a vector that follows the rules for vectors. We represent any force on a diagram by an arrow and its length is drawn proportional to magnitude of the force.

Newton's analysis of motion is summarized into three laws of motion. The first law of motion states that

> every body continues in its state of rest or of uniform speed in a straight line unless it is compelled to change that state by forces acting on it.

The tendency of a body to maintain its state of rest or of uniform motion in a straight line is called inertia. As a result, the first law of motion is often called the law of inertia.

2 - 2 The Second Law of Motion

Newton's second law makes use of the concept of mass. We often use the term mass as a synonym for quantity of matter. This notion of the mass of a body is not very precise because the concept "quantity of matter" is itself not well defined. More precisely we can say that mass is a measure of the inertia of a body. The more mass a body has the harder it is to change its state of motion - it is harder to start it moving from rest or to stop it when it is moving. To quantify the concept of mass we must define a standard. In SI units the unit of mass is the kilogram (kg). The actual standard is a particular platinium-iridium cylinder kept at the International Bureau of Weights a Measures, whose mass is precisely one kilogram.

From the first law of motion we know that if no net force is acting on a body it remains at rest, or if it is moving it continues moving with constant speed in a straight line. But if a force starts to act on a body, its velocity changes. Thus a net force gives rise to acceleration. The question of the relationship between acceleration and force can be answered from ordinary experience. If we double the force which acts on a body, the acceleration doubles; if we triple the force, the acceleration is tripled, and so on. So, the acceleration of a body is directly proportional to the net force acting on it. But the acceleration depends on the mass of the body as well. For a given net force we can say that the greater the mass, the less the acceleration. Thus, the acceleration of a body is inversely proportional to its mass. These relationships can be summarized as follows:

> the acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass. The direction of the acceleration is in the direction of the applied net force.

This is called the second law of motion.

So, we can write

$$\vec{a} \approx \frac{\vec{F}}{m} ,$$

where $\vec{a}$ stands for acceleration, m for the mass and $\vec{F}$ for the net force. By net force we mean the vector sum of all forces acting on the body. The choice of a constant can be arbitrary since we are relating quantities with different units. Rearranging, we have the familiar statement of the second law in form

$$\vec{F} = m\,\vec{a} . \qquad (2\text{-}1)$$

This is a vector equation - both direction and magnitude must be equal on the two sides of the equation.

The unit of force is called newton (N). One newton is the force required to impart an acceleration of 1 m/s^2 to a mass of 1 kg. Thus, 1 N = 1 kg m/s^2.

2 - 3 The Third Law of Motion

The second law describes quantitatively how forces affect motion. But where, we may ask, do forces come from. Observations suggest that a force applied to any object is always applied by another object. This is the essence of the third law of motion:

> Whenever one object exerts a force on a second object, the second exerts an equal and apposite force on the first.

This law is sometimes called as "to every action there is an equal and opposite reaction". It is very important to remember that the action force and the reaction force are acting on different objects.

2 - 4 Weight - the Force of Gravity

It is known that objects dropped near the surface of the earth will fall with the same acceleration $\vec{g}$, if air resistance can be neglected. The force that gives rise to this acceleration is called the force of gravity.

If we apply the second law to the force of gravity we can write

$$\vec{F}_g = m\,\vec{g} . \qquad (2\text{-}2)$$

The direction of this force is down toward the center of the earth. This force of gravity is being also called weight of body.

In SI units, $g = 9{,}80\ m/s^2$, so the weight of 1 kg mass is 9,80 N. The value of g varies very slightly at different places on the earth's surface. On the moon, on other planets, or in space, the weight of given mass will be different.

The terms mass and weight are often confused with one another, but it is important to distinguish between them. Mass is property of a body itself. Weight, on the other hand, is a force, the force of gravity acting on a body.

2 - 5 Normal Force

The force of gravity acts on an object when it is falling. When an object is at rest on the earth the gravitational force does not disappear. From the second law of motion the result force on an object at rest is zero. There must be another force on the object to balance the gravitational force.

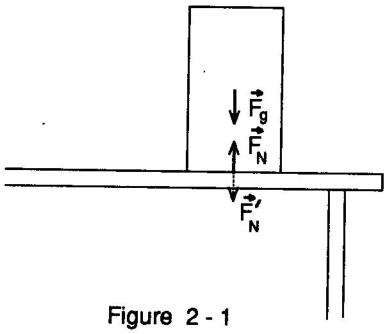

Figure 2 - 1

For an object resting on a table, the table exerts this upward force. The table is compressed slightly by the object and due to its elasticity it pushes up on the object. The force exerted by the table is often called a contact force, since it occurs when two object are in contact. When a contact force acts normal (means perpendicular) to the common surface of contact, it is usually referred to as the normal force $\vec{F}_N$. Now we have two forces which are both acting on the object, which remains at rest. So the vector sum of these two forces must be zero. Hence $\vec{F}_g$ and $\vec{F}_N$ must be of equal magnitude and opposite direction. But we must remember that upward force $\vec{F}_N$ on the object is exerted by the table. The reaction to this force is a force $\vec{F}'_N$ exerted by the object on the table. This force exerted on the table by the object is the reaction force to $\vec{F}_N$. We can also say the reverse: the force $\vec{F}_N$ on the object exterted by the table is the reaction to the force $\vec{F}'_N$ exerted on the table by the object.

Example : A box of mass m rests on the frictionless horizontal surface of a table.

a) Determine the weight of the box and the normal force acting on it.
b) A person pushes down on the box with a force F_p ; determine again the box's weight and the normal force acting on it.
c) If a person pulls upward on the box with a force F_p less than weight, what now is the box's weight and the normal force on it?

Solution:

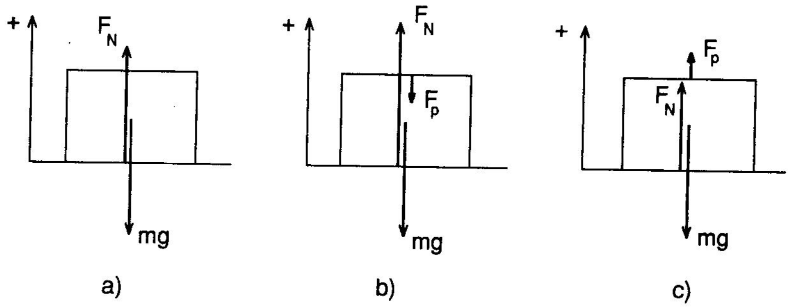

Figure 2-2

a) The weight of the box is mg. If we have chosen the upward direction as positive the net force on the box is $F = F_N - mg$, where F_N is the normal force exerted upward on it by the table. Since the box is at rest, the net force on it must be zero, so $F = ma$ and $a = 0$, thus

$$F_N - mg = 0$$

and

$$F_N = mg.$$

b) The weight of the box is still mg. By the Fig. 2-2b the net force is (upward direction is positive)

$$F = F_N - mg - F_p .$$

The net force must be equal to zero since the box remains at rest ($F = ma$ and $a = 0$). Thus

$$F_N - mg - F_p = 0 ,$$

so

$$F_N = mg + F_p .$$

c) The box's weight is still mg. The box does not move since the upward force is less than the weight. The net force is equal (upward direction is positive)

$$F = F_N + F_p - mg$$

and again it must be equal to zero, so

$$F_N + F_p - mg = 0$$

and

$$F_N = mg - F_p .$$

2-6 Applications of Laws of Motion

The second law of motion tells us that the acceleration of an object is proportional to the net force acting on the object. The net force here means the vector sum of all forces acting on the object. Several useful examples of application of the laws of motion follow.

Example 1: Calculate the sum of two force vectors acting on the small boat in Fig. 2-3a.

Solution: After resolving these two vectors (Fig. 2-3b) we can express the components of $\vec{F}_1$ and $\vec{F}_2$ as follows:

$$F_{1x} = F_1 \cos \alpha$$
$$F_{1y} = F_1 \sin \alpha$$
$$F_{2x} = F_2 \cos \beta$$
$$F_{2y} = F_2 \sin \beta$$

where F_1, F_2 are the magnitudes of the vectors $\vec{F}_1$, $\vec{F}_2$, respectively.

As the angle β has a negative sign, the component F_{2y} is negative and it points along the negative y axis.

Components of the resultant force are given as the sum of the components of the forces $\vec{F}_1$ and $\vec{F}_2$ (see Fig. 2-3c)

$$F_{Rx} = F_{1x} + F_{2x}$$
$$F_{Ry} = F_{1y} + F_{2y}$$

To find the magnitude of the resultant force, we use the Pythagorean theorem, so

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

and for the angle φ which the force $\vec{F}_R$ makes with the x axis we use

$$\operatorname{tg} \varphi = \frac{F_{Ry}}{F_{Rx}} .$$

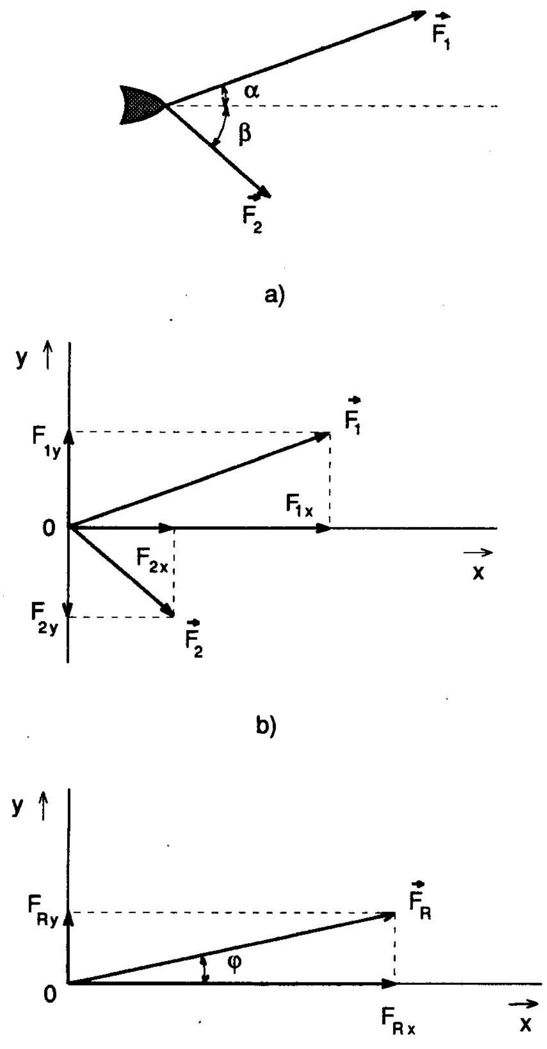

Figure 2 - 3

Example 2: Calculate the force required to accelerate the 20 kg cart from rest to 0,5/s in 2 s (friction is negligible).

Solution: There are three forces acting on the cart. The forward pushing force $\vec{F}_p$ exerted by the person, the downward force of gravity $\vec{F}_g$ and the upward force $\vec{F}_N$ exerted by the floor (which is the reaction to the force of the cart pushing down on the floor). The sum of both vertical forces $\vec{F}_g$ and $\vec{F}_N$ must be zero; if it did not the cart would accelerate vertically. So $|\vec{F}_N| = |\vec{F}_g| = mg = 196$ N. Then the net force on the cart is simply $\vec{F}_p$.

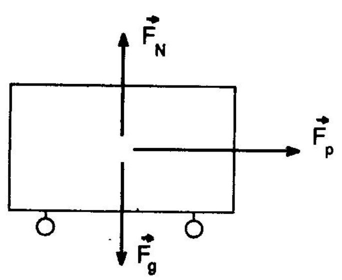

Figure 2 - 4

To calculate how large $\vec{F}_p$ must be, we first calculate the acceleration required:

$$a = \frac{v}{t} = \frac{0{,}5}{2} = 0{,}25 \text{ m/s}^2.$$

So, the magnitude of the force exerted by the person must be

$$F_p = ma = 20.0{,}25 = 5N .$$

Example 3 : A box of the mass m is being pulled by a person along the surface of a table with a force $\vec{F}_p$. The force is applied at an angle α (Fig. 2-5). The friction is assumed to be neglected. Calculate

a) the acceleration of the box,

b) the magnitude of the upward force $\vec{F}_N$ exerted by the table on the box.

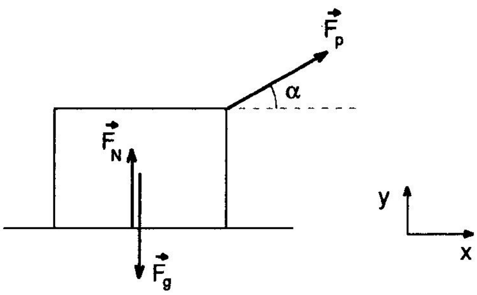

Figure 2-5

Solution : We resolve all forces into components:

$$\vec{F}_p = (F_p \cos\alpha,\ F_p \sin\alpha)$$
$$\vec{F}_g = (0,\ -mg)$$
$$\vec{F}_N = (0,\ F_N)$$

In the horizontal (x) direction, $\vec{F}_N$ and $\vec{F}_g$ have zero components, thus,

$$F_{px} = ma_x ,$$

so

$$a_x = \frac{F_{px}}{m} = \frac{F_p \cos\alpha}{m} .$$

In the vertical direction we have

$$ma_y = F_{Ny} + F_{py} + F_{gy} .$$

We know $a_y = 0$ since the box does not move vertically. Then ($F_{Ny} = F_N$, $F_{gy} = - mg$, $F_{py} = F_p \sin\alpha$)

$$0 = F_N + F_p \sin\alpha - mg$$

and

$$F_N = mg - F_p \sin\alpha .$$

Notice that F_N is less than F_g. The ground does not push against the full weight of the box since part of the pull force exerted by the person is in the upward direction.

Example 4 : Two boxes connected by a lightweight cord are resting on a table. The boxes have masses m_1 and m_2. A horizontal force of F_p is applied to the right box as shown in Fig. 2-6 (friction is neglected). Find:

a) the acceleration of boxes,

b) the tension in the cord.

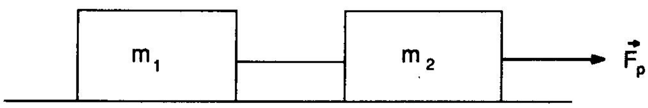

Figure 2-6

Solution : We draw the force diagram for each of the boxes (Fig. 2-7).
We can neglect the cord mass relative to the mass of the

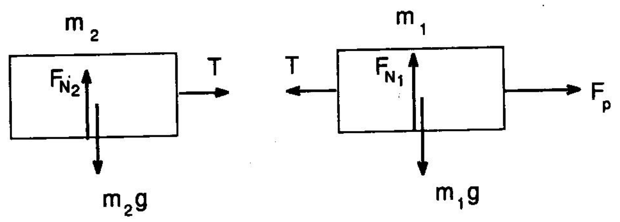

Figure 2 - 7

boxes. The force F_p acts on box m_1 ; box m_1 exerts a force T on the connecting cord and the cord exerts a force - T back on box m_1 (the third law of motion). Because the cord is considered to be massless, the tension at each end is the same. So the cord exerts a force T on the box m_2. The acceleration of both boxes is the same. For the horizontal motion we have:

for box m_1 $\qquad m_1 a = F_p - T$

for box m_2 $\qquad m_2 a = T$

Hence

$$m_2 a = F_p - m_1 a$$

and

$$a = \frac{F_p}{m_1 + m_2} .$$

For the tension T we have $T = m_2 a$ or $T = F_p - m_1 a$.

E x a m p l e 5 : Suppose the cord in Example 4 is a heavy rope of mass m . Calculate the acceleration of each box and the tension in the rope.

S o l u t i o n : Since the cord has mass, the product ma will not be zero, so the forces (tension) at either end will not be the same (Fig. 2-8). Therefore

$$T_1 - T_2 = ma . \qquad (a)$$

T_1 is the magnitude of the force that box m_1 exerts on the cord and that the cord exerts back on box m_1. T_2 is the magnitude of the force the cord exerts on box m_2 and that box m_2 exerts back on the cord.

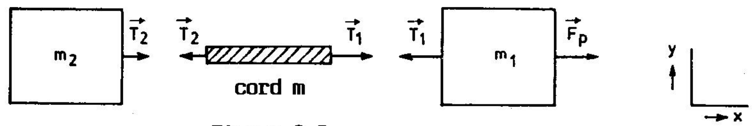

Figure 2-8

Fig. 2-8

For box m_1 we have

$$F_p - T_1 = m_1 a \qquad (b)$$

and for box m_2

$$T_2 = m_2 a. \qquad (c)$$

We now have obtained three equations (a, b, c) in three unknowns T_1, T_2 and a .

E x a m p l e 6 : Suppose two different boxes ($m_2 > m_1$) are placed with the cord joining them hanging over a frictionless massless pulley as in Fig. 2-9. We assume the cord is massless. Calculate the acceleration of boxes and the tension in the cord.

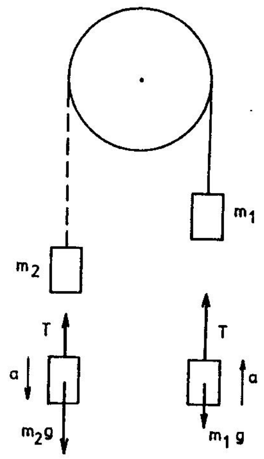

Figure 2-9

S o l u t i o n : Since the cord is massless the tension T is the same at its two ends. As the box m_2 is the heavier, it accelerates downward and box m_1 accelerates upward.

From Fig. 2-9 it is clear that

$$m_1 g < T < m_2 g \ .$$

To find its value we write the second law of motion for each box, taking the upward direction as positive for both boxes:

$$T - m_1 g = m_1 a \ ,$$

$$T - m_2 g = - m_2 a .$$

We subtract the second equation from the first to get

$$(m_2 - m_1) g = (m_1 + m_2) a$$

and solving this for a :

$$a = \frac{m_2 - m_1}{m_1 + m_2} g \ .$$

The tension T we can get from either of the two equation above

$$T = (a + g) m_1 \ ,$$

$$T = (g - a) m_2 \ .$$

2 - 7 Friction Force

Until now we have ignored friction which must be however taken into account in most practical problems. First we will be concerned with sliding friction that is usually called kinetic friction.

When a body is in motion along a rough surface, the force of kinetic friction acts opposite to the direction of the body's motion. The magnitude of the force of kinetic friction depends on the nature of the two sliding surfaces. For given surfaces, it is proportional to the normal force between the two surfaces, which is the force that either object exerts on the other, perpendicular to their common surface of contact. It does not depend on the total surface area of contact.

Hence, for friction we can write

$$F_f = \mu_k F_N \ . \qquad (2\text{-}3)$$

This is a relation between the magnitude of the friction force F_f, which acts parallel to the two surfaces, and the magnitude of the normal force F_N, which acts perpendicular to the surfaces. It is not vector equation since the two forces are perpendicular to one another. The term μ_k is called the coefficient of kinetic friction.

Now suppose an object such as a desk is resting on a horizontal floor. If no horizontal is exerted on the desk, there also is no friction force. But now, suppose we try to push the desk, but it does not move; we are exerting a horizontal force, but the desk is not moving, so there must be another force on the desk keeping it from moving. This is the force of static friction exerted by the floor on the desk.

If we push with a greater force enough, the desk will finally start to move. At this moment we have exceed the maximum force of static friction, which is given

$$F_f = \mu_s F_N , \qquad (2-4)$$

where μ_s is the coefficient of static friction.

Since the force of static friction varies from zero to this maximum value, we can write for it

$$F_f \leq \mu_s F_N .$$

μ_s can never be less μ_k therefore it is often easier to keep a heavy object moving than it is to start it moving.

See and try to understand Fig. 2-10. The graph describes the dependence of the magnitude of the force of friction as a function of the external force applied to a body initially at rest. As the applied force is increased, the force of static friction increases linearly until the applied force equals $\mu_s F_N$. If the applied force increases further, the body will begin to move and the friction force drops to a constant value characteristic of kinetic friction.

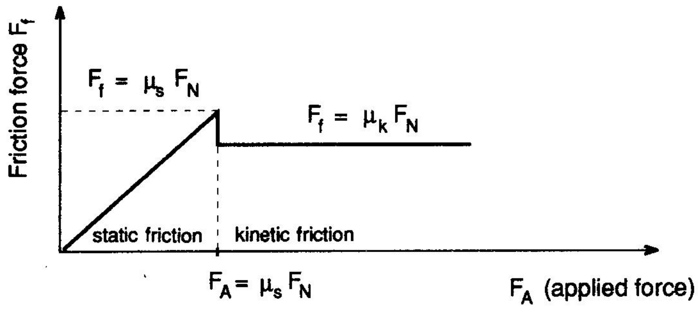

Figure 2 - 10

Now we look at some examples involving kinetic friction. Note that both the normal force and the friction force are forces exerted by one surface on the other; one is perpendicular to the contact surfaces (the normal force), and the other is parallel (the friction force).

Example: A box of mass m is pulled along a horizontal surface by a force F_p which is applied at an angle α. This is like example in the preceding section except now there is friction (see Fig. 2-11). Calculate the acceleration.

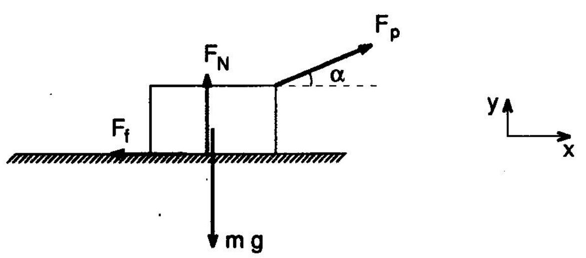

Figure 2 - 11

Solution: The force of kinetic friction opposes the direction of motion and is parallel to the surfaces of contact. If $F_{py} = F_p \sin \alpha$ is less than the weight of the box mg, so there is no motion in the vertical direction (y). Taking y as positive upward, we have

$$F_N - mg + F_p \sin \alpha = ma_y$$

or

$$F_N - mg + F_p \sin \alpha = 0$$

and

$$F_N = mg - F_p \sin \alpha .$$

In the horizontal direction (x) - positive to the right - we have

$$F_{px} - F_f = ma_x$$

and

$$a_x = \frac{F_p \cos \alpha - \mu_k F_N}{m} .$$

2 - 8 Dynamics of Circular Motion

In Chapter 1 we discussed the kinematics of a particle moving in a circular path. We now study the dynamics of circular motion. We saw that a particle revolving in a circle of radius r with uniform speed v undergoes a radial acceleration

$$a_R = \frac{v^2}{r} , \qquad (2\text{-}5)$$

or

$$a_R = \omega^2 r ,$$

where $\omega = \frac{v}{r}$ is the angular velocity of the particle.

This acceleration a_R is called radial or centripetal acceleration because it is directed toward the center of the circle. Although the magnitude of the acceleration (radial) vector in uniform circular motion is constant its direction is changing. Hence the acceleration vector $\vec{a}_R$ is a variable acceleration vector. The direction of vector $\vec{a}_R$ is always perpendicular to the velocity vector $\vec{v}$.

For object moving uniformly in a circle a force is necessary to give it a centripetal acceleration. The magnitude of the required force can be calculated using the second law of motion F = ma, where we use the value of the centripetal acceleration (see Eq. 2-5):

$$F = ma_R = m \frac{v^2}{r} = m \omega^2 r . \qquad (2\text{-}6)$$

Since a_R is directed along radius toward the center of the circle at any moment, this force must be directed toward the center of the circle, too. This force we called a centripetal force. This force must be applied by some object. For example, when a person swings a ball on the end of a string in a circle, the person exerts the force on the ball.

There is misconception that an object moving in a circle has an outward force acting on it, a so-called centrifugal force. To keep the ball moving in a circle, the person pulls on the ball. The ball, then, exerts an equal and opposite force on the your hand (the third law of motion), and this is the force your hand feels. The force on the ball is the force exerted on it by you. Centrifugal force does not act one the ball. If a centrifugal force were acting, the ball would fly outward; but it flies off tangentially in the direction of the velocity it had at the moment it was released.

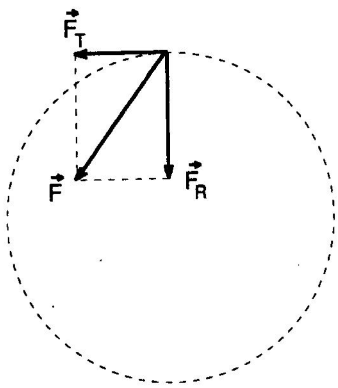

Figure 2 - 12

Circular motion at constant speed occurs when the force on an object is exerted toward the center of the circle.

If the force is not directed toward the center but acts at an angle as shown in Fig. 2-12, then the force has two components. The component $\vec{F}_R$ directed toward the center of the circle gives rise to the centripetal acceleration a_R, and keeps the object moving in a circle. The component $\vec{F}_T$, tangential to the circle, acts to increase (or decrease) the speed and thus gives rise to the tangential acceleration.

Example: A particle of mass m suspended by a cord of length L revolves in a circle of radius $r = L \sin \varphi$, where φ is the angle the string makes with the vertical (see Fig. 2-13). Calculate the speed and period of the motion.

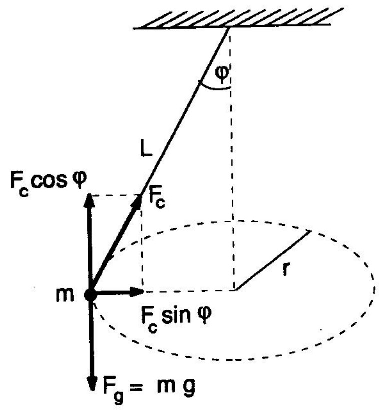

Figure 2 - 13

Solution: The forces acting on the mass m are its weight $F_g = mg$ and the force exerted by the cord F_c which has horizontal and vertical components of magnitude $F_c \sin \varphi$ and $F_c \cos \varphi$, respectively. Let us apply the second law of motion to the horizontal and vertical direction.

In the vertical direction there is no motion of a particle, so the acceleration is zero and we can write

$$F_c \cos \varphi - mg = 0 . \qquad (2\text{-}7)$$

In the horizontal direction there is only one force of magnitude $F_c \sin \varphi$, that acts toward the center of the circle and gives rise to the centripetal acceleration v^2/r, so

$$F_c \sin \varphi = m \frac{v^2}{r} .$$

From the last equation we have (using Eq. 2-7 for F_c)

$$v = \sqrt{\frac{rF_c \sin \varphi}{m}} = \sqrt{\frac{r}{m}\left(\frac{mg}{\cos \varphi}\right) \sin \varphi} .$$

Since $r = L \sin \varphi$, we have

$$v = \sqrt{\frac{L g \sin^2 \varphi}{\cos \varphi}} .$$

The period T is the time required to make one revolution of distance $2\pi r = 2\pi L \sin \varphi$. Thus

$$T = \frac{2\pi L \sin \varphi}{v} = 2\pi L \sin \varphi \sqrt{\frac{\cos \varphi}{Lg \sin^2 \varphi}} = 2\pi \sqrt{\frac{L \cos \varphi}{g}} .$$