Task 2 - Measurement of the acceleration due to the gravity with reversible pendulum and study of the gravitational field

2.1 Measurement task

1. Specify the size of gravitational acceleration for Prague by reverse pendulum.
2. Specify the error of the measurement of gravitational acceleration.
3. Make a final correction value for the period of reverse pendulum and compare corrected value with the measured value.
4. Make a graph of dependency on lens position of τ_{0d} and τ_{0h}.

2.2 Methods of measurement

1. Turn on the clocks and the second switch set to position "**START**".

2. Hang the pendulum in the down position with the lens and set the shortest distance from the edge. Do not forget to lightly tighten insurance mother. Pendulum deflects from the equilibrium to the left fence without touching it.

3. As next step press "**NULOVÁNÍ**". Oscillation counter will set to zero and from the first pass by equilibrium start counting oscillation and time. After each 100 oscillation time will freeze on the display for 5 sec.

4. Take off time $100\tau_{0d}$. Hang the pendulum with the lens on the top and again deflect it to the left and take off time $100\tau_{0h}$.

5. Enlarge the distance of the lens from the edge by 2 revolutions (pitch is 1 mm) and repeat the measurement like in points 2., 3. a 4. Measured values set to the graph as a function of lens location of reverse pendulum.

6. Continue in measuring until the curves expressing dependency τ_{0d} and τ_{0h} of the lens position will cross.

7. If is the lens in position of intersection of both curves, make another measurement for time τ_0 for 500 swings.

8. Set the average from $500\tau_{0d}$ and $500\tau_{0h}$and calculate for it the value of gravitational acceleration.

9. Estimate the precision timing and accuracy of measurement and from this values calculate the precision of measurement device.

10. Obtained values compare with the table of values for Prague.

2.3 Used tools

Reverse pendulum
Hinge with optical sensor
Counter swing chronograph
Tape measure

2.4 Tables of measurements and processing results

Lens distance [mm]	Lens position			
	down	up	down	up
	$100\tau_{0d}$ [s]	$100\tau_{0h}$ [s]	$500\tau_{0d}$ [s]	$500\tau_{0h}$ [s]
4	77,59	77,51		
5	77,61	77,65	388,4	388,54
6	77,66	77,84		
7	77,76	78,33		
8	77,81	78,61		
9	77,87	78,84		
12	78,03	79,63		

Determine the mean time $500\tau_{0d}$ and $500\tau_{0h}$

$$500\tau_0 = \frac{500\tau_{0d} + 500\tau_{0h}}{2} = \frac{388{,}40 + 388{,}54}{2} = 388{,}47\ s \Rightarrow \tau_0 = \frac{388{,}47}{500} = 776{,}94 \cdot 10^{-3}\ s$$

Calculation of the value of gravity g for $500\tau_0$:

$$g = \frac{\pi^2 \cdot L}{\tau_0^2} = \frac{\pi^2 \cdot 0{,}596}{0{,}77694^2} = 9{,}745 \quad m \cdot s^{-2}$$

As precondition that the digital stopwatch measured precisely on displayed number places was accuracy of time measurement $\vartheta(t) = \pm\ 0{,}01 s$.
Accuracy of measurement of distance edges was $\vartheta(L) = \pm 0{,}001\ m$ and due to rounding to 2 decimal places we can ignore it.

The deviation of measured values from the average is:

$$\tau_{0d} = \frac{388{,}40}{500} = 776{,}8.10^{-3}\ s$$

$$\tau_{0h} = \frac{388{,}54}{500} = 777{,}08.10^{-3}\ s$$

$$\Delta\tau_{od} = \tau_{od} - \tau_o = -140.10^{-6}\ s$$

$$\Delta\tau_{oh} = \tau_{oh} - \tau_o = 140.10^{-6}\ s$$

Probable result error with uncertainties of measure $\vartheta(V)$ we calculate from relationship :

$$\vartheta(V)=\sqrt{\left(\frac{\partial f}{\partial x}\right)^2\vartheta^2(x)+\left(\frac{\partial f}{\partial y}\right)^2\vartheta^2(y)+\left(\frac{\partial f}{\partial z}\right)^2\vartheta^2(z)+\ldots},$$

where $\vartheta(x),\vartheta(y),\vartheta(z)$, ... are probable error values x, y, z ... and expressions $\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z},\ldots$ are first partial derivatives functions *f(x,y,z,...)*.

Probable error of *g* is:

$$\vartheta(g)=\sqrt{\left(-\frac{2\pi^2L}{\tau_0^3}\cdot\vartheta(\tau_0)\right)^2+\left(\frac{\pi^2\vartheta(L)}{\tau_0^2}\right)^2}$$

$$\vartheta(g)=\sqrt{\left(-\frac{11{,}753}{0{,}467}\cdot 0{,}01\right)^2+\left(\frac{0{,}0099}{0{,}602}\right)^2}=0{,}25\ ms^{-2}$$

Gravity acceleration g is: $g=(9{,}773\pm 0{,}25)\ ms^{-2}$

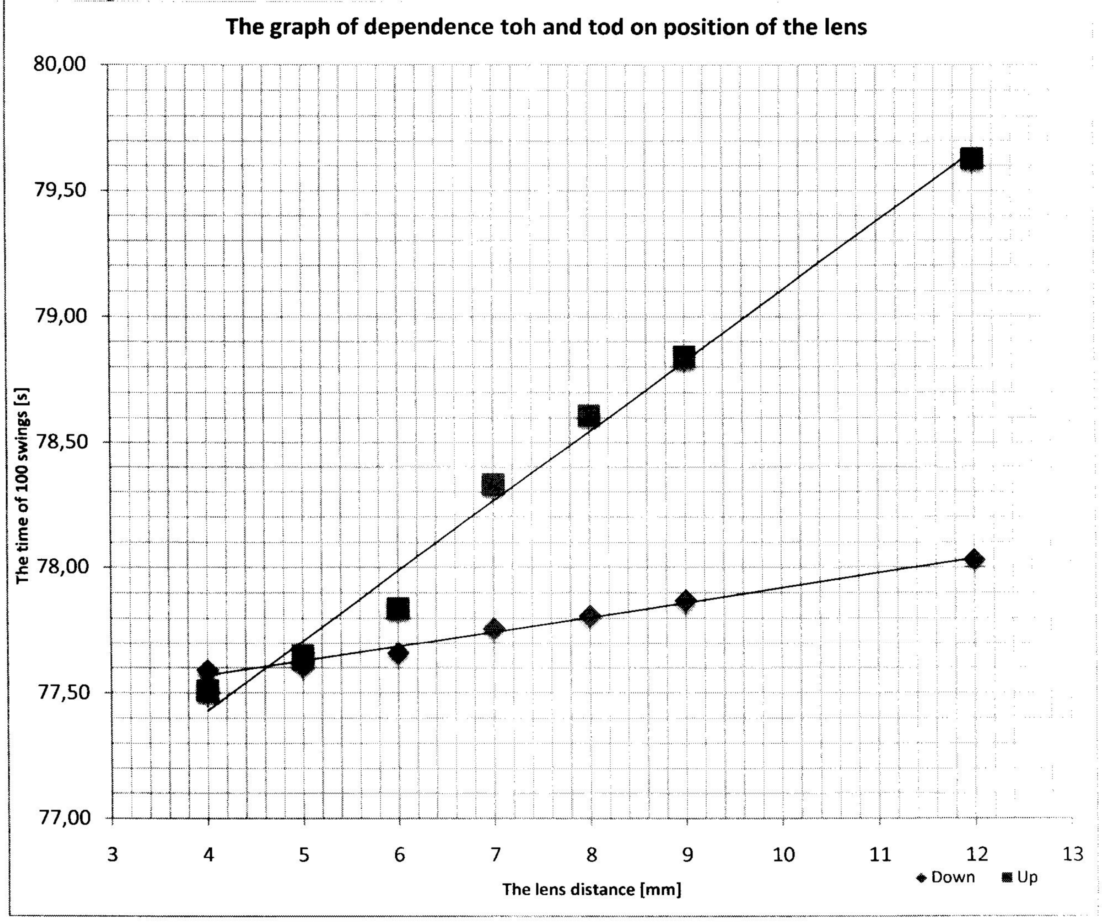
2.5 Results & conclusion

Probable error of measuring time: $\vartheta(t)=\pm 0{,}01\ s$
Probable error of accuracy of the distance of edges: $\vartheta(L)=\pm 0{,}001\ m$

The value of gravitational acceleration: $g=(9{,}773\pm 0{,}014)\ ms^{-2}$

After the evaluation of measurement we define the gravitation acceleration g for $g=(9{,}75\pm 0{,}25)\ ms^{-2}$, which is nearly to gravitational acceleration for Prague which is referenced to the table value 9,81040 ms^{-2} (difference from the table value is 0,71%). For the condition that the digital stopwatch in counter measured precisely on the number of displayed numbers was accuracy of time measurement $\vartheta(t)=\pm 0{,}01s$. The accuracy of the distance of edges was $\vartheta(L)=\pm 0{,}001\ m$ and due to rounding to 2 decimal places we can ignore it.

2.6 Graphs



The graph of dependence toh and tod on position of the lens
80,00
79,50
79,00
78,50
78,00
77,50
77,00
The time of 100 swings [s]
3
4
5
6
7
8
9
10
11
12
13
The lens distance [mm]
Down
Up