4. GEOMETRICAL OPTICS

This chapter is devoted to the ray model of light. The straight-line paths that light follows are called **light rays**. The ray model of light is very successful in dealing with many aspects of light such as reflection, refraction, and the formation of images by mirrors and lenses. Because these explanation involve straight-line rays at various angles, this subject is referred to as geometrical optics.

As for the **speed of light** we can say that the **accepted value** today for the speed of light ,c, **in vacuum** is

$$c = 2.99792458 \times 10^8\ m/s.$$

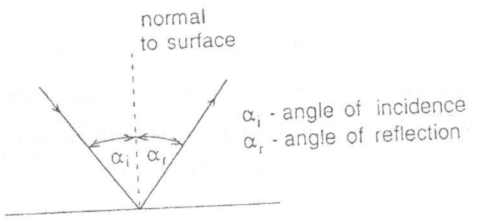
We usually round this value off to $3.00 \times 10^8\ m/s$ when extremely precise results are not required. In air, the speed is only slight less. In other transparent materials such as glass and water, the speed is always less than that in vacuum.

The ratio of the speed of light in vacuum c to the speed v in a given material is called the **index of refraction** n of that material

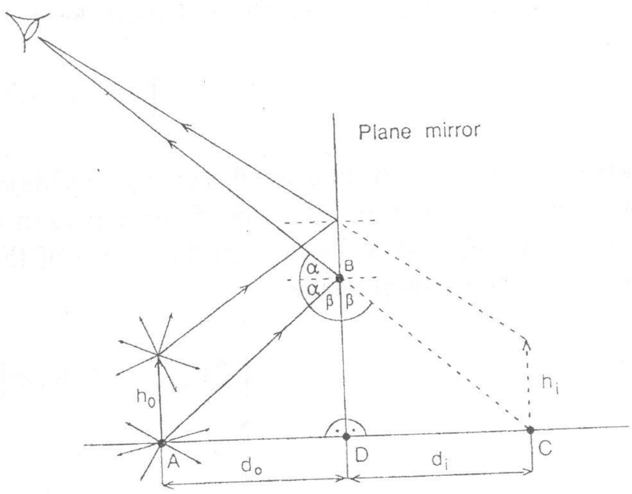
$$n = \frac{c}{v}.$$

4.1 Reflection; Plane Mirror

When light strikes the surface of an object, some of light is reflected and the rest is either absorbed by the object (and transformed to heat) or, if the object is transparent (like glass or water), part of it is transmitted through.



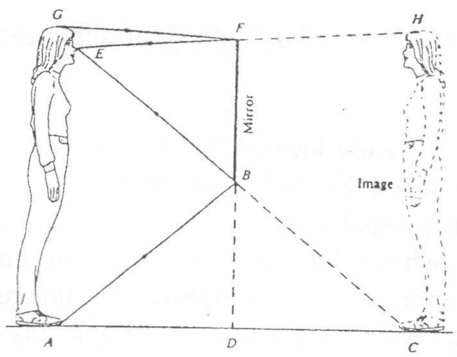
We shall be now interested in a reflection. From the Fig. we define the **angle of incidence** α_i to be the angle an incident ray makes with the normal to the surface and the **angle of reflection** α_r to be the angle the reflected ray makes with the normal. For flat surface the incident and reflected rays lie in the same plane with the normal, called the incident plane, and **the angle of incidence equals the angle of reflection** - this is called the **law of reflection**.



The following figure shows how an image is formed by a plane mirror. Two triangles

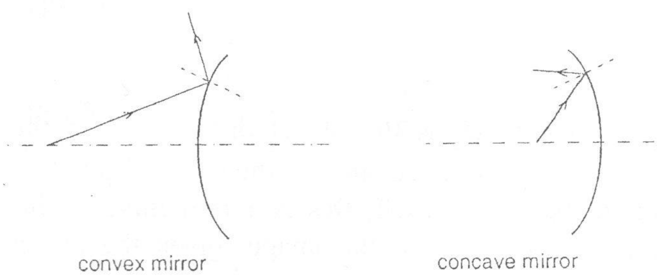
ABD and CDB are congruent and the lengths AD = CD. That is, the image is as far behind the mirror as the object is in front. We say that the image distance d_i equals the object distance d_o , and we also see that the height of the image is the same as that of the object. Since the rays do not actually pass through the image, a piece of white paper or film placed at the image would not detect the image. Such the image is called a **virtual image**. In a **real image** the light does pass through the image and such the image can appear on paper or film placed at the image position.

Problem 4-1. A woman 1.60 m tall stands in front of a vertical mirror. What is the minimum height of the mirror and how must its lower edge be above the floor if she is to be able to see her whole body? Assume her eyes are 10 cm below the top of her head.



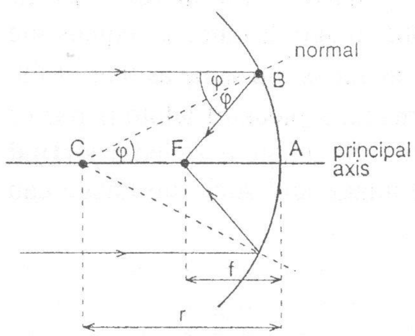
Solution: First we consider the ray from the toe, AB, which upon reflection becomes BE and enters the eye E. Since light enters the eye from point A (the toes) after reflecting at B, the mirror needs to extend no lower than B. Because the angle of reflection equals the angle of incidence, the height BD is half of the height AE. Since AE = 1.60 m - 0.10 m = 1.50m, BD = 0.75 m. If the woman is to see the top of her head, the top edge of the mirror only needs to reach point F, which is 5 cm below the top of her head (half of GE = 10 cm). Thus DF = 1.55 m and the mirror need be only 1.55 m - 0.75 m = 0.80 m high and its bottom edge must be 0.75 m above the floor.
We may say generally that a mirror need be only half as tall as a person for that person to see all of himself.

4.2 Spherical Mirrors



Reflecting surface do not have to be flat. The most common curved mirrors are spherical. A spherical mirror is called **convex** if the reflection takes place on the outer surface of the spherical shape and it is called **concave** if the reflecting surface is on the inner surface of the sphere.

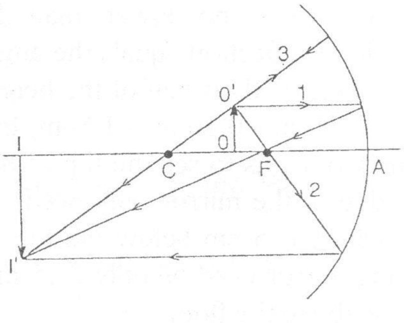
Generally, a spherical mirror does not make as sharp an image as a plane mirror does. However, if the reflecting surface of a mirror has a width that is small compared to its radius of curvature, the rays parallel to the principal axis make only a small angle upon reflection, then the rays will cross each other at nearly a simple point, or **focus F**, or the



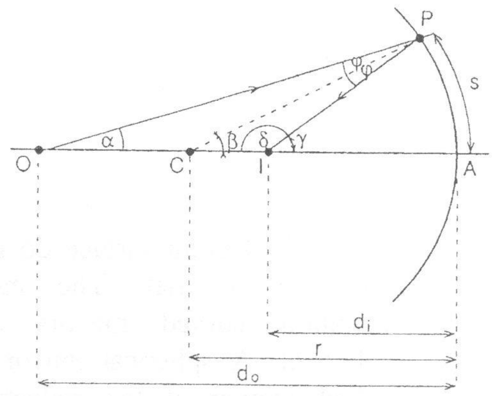
focal point of the mirror. The **principal axis** is defined as the straight line perpendicular to the curved surface at it center (line CA in Fig.) The distance FA is called the **focal length, f,** of the mirror. Another way of defining the focal point is to say that it is the **image point for an object infinitely far away** along the principal axis. The point C is the center of curvature of the mirror. The line CB is equal to r, the radius of curvature. **For the mirror that is small compared to its radius of curvature the focal length in this approximation is half the radius of curvature**

$$\mathbf{f = r/2}$$

and the image is sharp.
If the larger the mirror is, the worse this approximation is. This "defect" of spherical mirrors is called **spherical aberration**.



We have already known that for an object at infinity the image is located at the focal point of a mirror if that is small compared to its radius of curvature. But where does the image lie for an object not at infinity? To determine the image position is simplified if we use three simple rays (see Fig.). Ray 1 is drawn parallel to the axis; therefore it must pass through F. Ray 2 is drawn through F; therefore it must reflect so it is parallel to the axis. Ray 3 is drawn so that it passes through C, the center of curvature, and thus is along a radius of the spherical surface; so it is perpendicular to the mirror and thus will be reflected back on itself.



It can be derived analytically the equation called the **mirror equation**

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}$$

where d_0 is the object distance, d_i is the image distance, and f is the focal length.

Because the light actually passes through this image itself, this is a **real image**. This equation gives us a way of determining the position of the image, given the object position and the focal length (or radius of curvature) of the mirror.

The **lateral magnification, m,** of a mirror is defined as the height of the image, h_i , divided by the height of the object, h_0

$$m=\frac{h_i}{h_0}=-\frac{d_i}{d_0},$$

where the minus sign is inserted as a convention.

The conventions we use are: if h_0 is considered positive, h_i is positive if the image is upright and negative if inverted; d_i and d_0 are positive if image and object are on the reflecting side of the mirror, but if either image or object is behind the mirror the corresponding distance is negative. Thus the magnification is positive for an upright image and negative for an inverted image.

The analysis used for concave mirrors can be applied to convex mirrors. The mirror equation and the equation for the magnification hold for convex mirrors but the focal length f must be considered negative, as must the radius of curvature.

Problem 4-2. A 1.50-cm high object is placed 20 cm from a concave mirror whose radius of curvature is 30 cm. Determine the position of the image and its size.

Solution: The focal length f = r/2 = 15 cm. Since d_0 = 20 cm, we have

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0} = 0.0167\ cm^{-1}.$$

So the image is 60 cm from the mirror on the same side as the object. The lateral magnification is

$$m = -\frac{60}{20} = -3.$$

Therefore the image is (-3)(1.5) = -4.5 cm high, and is inverted.

Problem 4-3. A 1-cm high object is placed 10 cm from a concave mirror whose radius of curvature is 30 cm. Determine the position of the image and the lateral magnification.

Answer: $d_i = -30\ cm$. The minus sign means the image is behind the mirror. The lateral magnification is

$$m = -\frac{-30}{10} = +3.$$

So the image is 3 times larger than the object; the plus sign indicates that the image is upright.

Problem 4-4. A convex rearview car mirror has a radius of curvature of 40 cm. Determine the location of the image and its magnification for an object 10 m from the mirror.

Solution: With r = -40 cm , f = -20 cm, the mirror equation gives

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0} = -\frac{51}{10} m^{-1}.$$

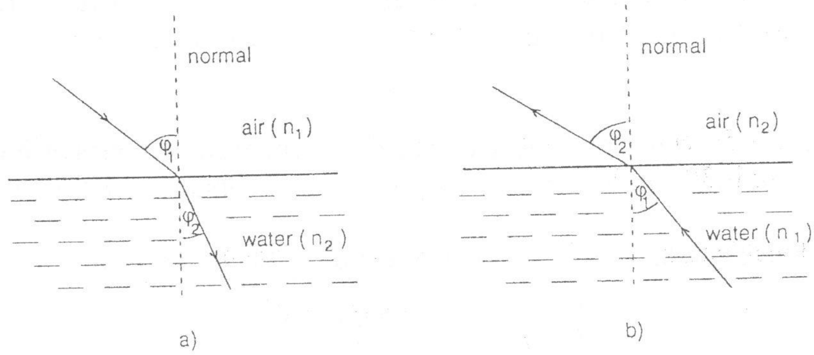
So $d_i = -0.196\, m$ or 19.6 cm behind the mirror. The lateral magnification is

$$m = -\frac{d_i}{d_0} = 0.0196$$

or 1/51; so the upright image is reduced by a factor of 51.

4.3 Refraction; Snell's Law

When light passes from one medium into another, part of the incident light is reflected at the boundary. The remainder passes into the new medium. If a ray of light is incident at an angle to the surface (other than perpendicular), the ray is bent as it enters the new medium. This bending is called **refraction**. The following figure shows a ray passing from air into water.



The angle φ_1 is the **angle of incidence** and φ_2 is the **angle of refraction**. In this case the ray bends toward the normal This is always the case when the ray enters a medium where the speed of light is less. If light travels from one medium into a second where its speed is greater, the ray bends away from the normal; for a ray traveling from water to air.

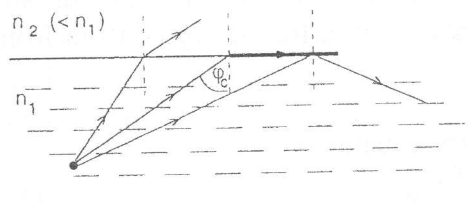
The angle of refraction depends on the speed of light in the two media and on the incident angle. A relation between φ_1 and φ_2 is known as Snell's law or the law of refraction. It is written as

$$n_1 \sin \varphi_1 = n_2 \sin \varphi_2 ,$$

φ_1 is the angle of incidence and φ_2 is the angle of refraction; n_1 *and* n_2 are the respective indices of refraction in the materials.

It is clear from Snell's law that if $n_2 > n_1$, then $\varphi_2 < \varphi_1$; that is, if light enters a medium where n is greater (and its speed less) then the ray is bent toward the normal. And if $n_2 < n_1$, then $\varphi_2 > \varphi_1$, so the ray bends away from the normal.

When light passes from one material into a second material where the index of refraction is less (say from water into air), the light bends away from the normal. At a particular incident angle, the angle of refraction will be 90^0. The incident angle at which this occurs is called the **critical angle** and from Snell's law it is given by



$$\sin \varphi_c = \frac{n_2}{n_1} \sin 90^0 = \frac{n_2}{n_1} .$$

For incident angles greater than φ_c Snell's law tell us that $\sin \varphi_2$ is greater than 1.00 . In this case there is no refracted ray at all, and all of the light is reflected. This is called **total reflection**. This total reflection can occur only when light strikes a boundary where the medium beyond has a lower index of refraction.

Problem 4-5. Light strikes a flat piece of glass with parallel faces at an incident angle of 60^0 . If the index of refraction of the glass is 1.5, what is the angle of refraction φ_a in the glass and what is the angle φ_b at which the ray emerges from the glass?

Solution: We assume the incident ray is in air so $n_1 = 1$ and $n_2 = 1.5$. Then,

$$\sin \varphi_a = \frac{1}{1.5} \sin 60^0 = 0.577,$$

so $\varphi_a = 35.2^0$.

Since the faces of the glass are parallel, the incident angle in this case is just φ_a . This time $n_1 = 1.5$ *and* $n_2 = 1$. Thus the angle φ_b is

$$\sin \varphi_b = \frac{1.5}{1} \sin \varphi_a$$

and $\varphi_b = 60^0$.

Problem 4-6. Our nearest star (other than the sun) is 4.2 light years away. That is, it takes 4.2 years for the light to reach earth. How far away is it in meters?

$[4 \times 10^{16}\, m]$

Problem 4-7. Suppose you are 60 cm from a plane mirror. What area of the mirror is used to reflect the rays entering one eye from a point on the tip of your nose if your pupil diameter is 5.5 mm?

$[5.9 \times 10^{-6}\, m^2]$

Problem 4-8. How far from a concave mirror (radius 24 cm) must an object be placed if its image is to be at infinity?

[12 cm]

Problem 4-9. The magnification of a convex mirror is 0.35 for objects 4 m away. What is the focal length of this mirror?

[-2.2 m]

Problem 4-10. What is the radius of a concave mirror that gives a magnification 1.6 of a face 30 cm from it?

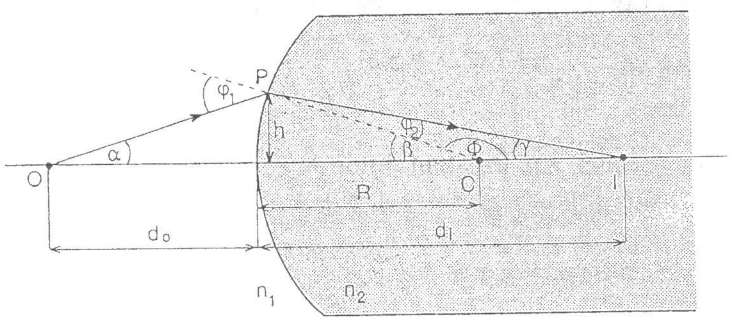
[80 cm]

Problem 4-11. A convex mirror has a radius of curvature of 20 cm. If a point sources is placed 14 cm away from the mirror, where is the image?

[-5.8 cm; the image is virtual]

4.4 Refraction at a Spherical Surface

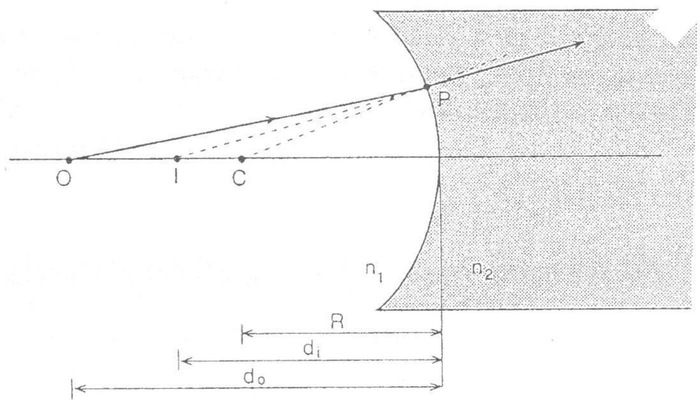
Let us consider an object which is located in a medium whose index of refraction is n_1. A ray from the object enters a medium whose index of refraction is n_2. The radius of curvature of the spherical boundary is R, and its center of curvature is at point C (see Fig.). It can be shown that all rays leaving point 0 will focused at a single point I, the image point, if we consider only rays (called paraxial rays) that make small angles φ_1, φ_2, α, β and γ. In this case it can be derived the equation



$$\frac{n_1}{d_0}+\frac{n_2}{d_i}=\frac{n_2-n_1}{R}.$$

The equation is also valid for a concave surface if we make the following conventions:

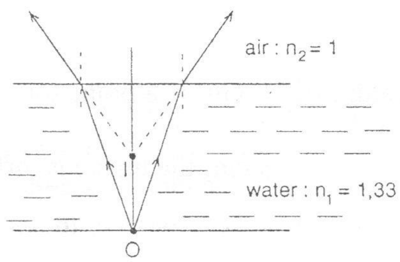
1. If the surface is convex (so the center of curvature C is on the side of the surface opposite to that from which the light comes), R is positive; if the surface is concave (C on the same side from which the light comes) R is negative.



2. The image distance d_i follows the same convention: positive if on the opposite side from where the light comes, negative if on the same side.

3. The object distance is positive if on the same side from which the light comes (this is normal case), otherwise it is negative.

For the case with a concave surface, both R and d_i are negative when used in the equation ; in this case the image is virtual.



Problem 4-12. A person looks vertically down into a 4-m deep lake. How deep does the lake appear to be?

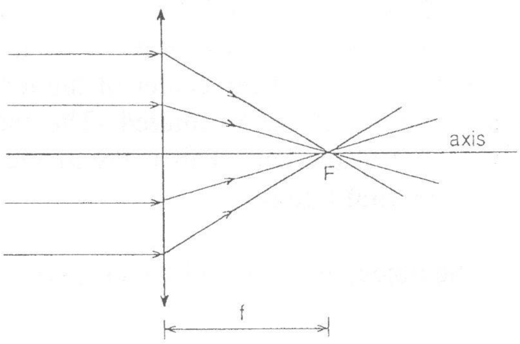
Solution: Point 0 represents a point on the lake bottom. The rays diverge and appear to come

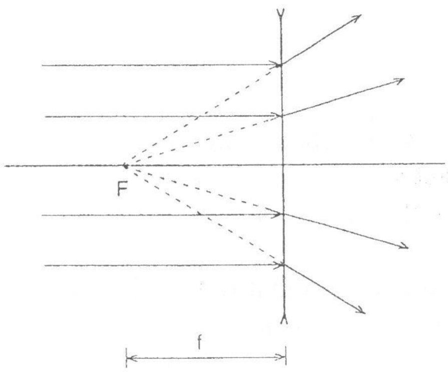
from point I, the image. We have $d_0 = 4m$ and, for a flat surface $R = \infty$.
Now we may write

$$\frac{1.33}{4} + \frac{1}{d_i} = 0$$

and thus $d_i = -3m$. So the lake appears to be only three-fourths as deep as it actually is. The minus sign tells us the image point I is on the same side of the surface as 0, and the image is virtual.

4.5 Thin Lenses





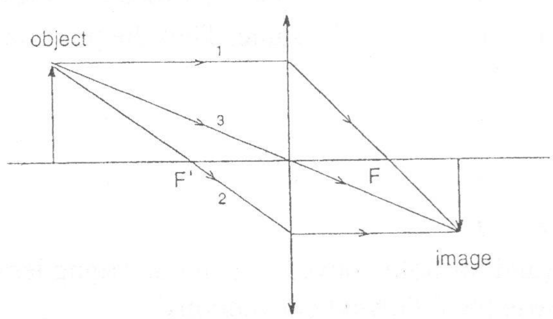
If the diameter of a lens is small compared to the radii of curvature of the two lens surface the lens is called a thin lens.

If rays parallel to the principal axis fall on a thin converging lens, they will all be focused to a point called the **focal point F**. We can also say that **the focal point is the image point for an object on the principal axis at infinity**. The distance of the focal point from the center of the lens is called the **focal length, f**.

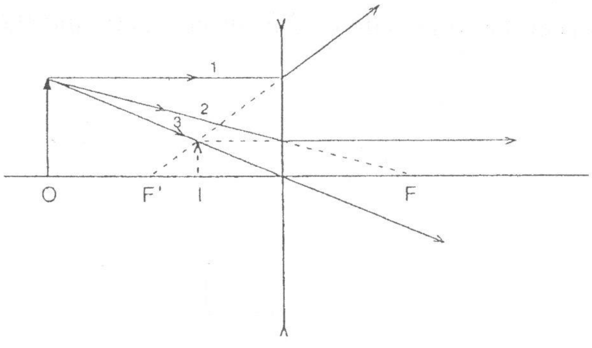
The plane perpendicular to the axis of the lens and passing through the focal point is called the **focal plane** of the lens. If parallel rays fall on a lens at an angle, they focus at another point placed in this focal.

The focal point F of a diverging lens is defined as that point from which refracted rays, originating from parallel incident rays, seem to emerge. And the distance from F to the lens is called the focal length, just as for a converging lens.

The image formed by a lens for a given object can be found by drawing three particular rays as indicated in the following figures.



For a converging lens the image position can be determine by drawing three rays. Ray 1 is drawn parallel to the axis; therefore it is refracted by the lens so it passes through the focal point F behind the lens. Ray 2 is drawn through the focal point F' on the same side of the lens as the object; it therefore emerges from the lens parallel to the axis. Ray 3 is directed toward the center of the lens where the two surfaces are essentially parallel to each other; this ray therefore emerges from the lens at the same angle as it entered. The image points for all other points on the object can be found similarly to determine the complete image of the object. Because the rays actually pass through the image, it is a **real image**.



For a diverging lens the image position can be determine by drawing the same three rays. Ray 1 is drawn parallel to the axis, but does not pass through the focal point F behind the lens, instead it seems to come from the focal point F' in front of the lens. Ray 2 is directed toward F and is refracted parallel by the lens. Ray 3 is again directed toward the center of the lens ; this ray therefore emerges from the lens at the same angle as it entered. The three refracted rays seem to emerge from a point on the left of the lens. This is the image, I. Since the rays do not pass through the image, it is a **virtual image**.

An equation that relates the image distance to the object distance and the properties of a thin lens has form

$$\frac{1}{d_0}+\frac{1}{d_i}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \qquad (1)$$

This equation relates the object distance d_0 to the image distance d_i (the distance from the lens of the final image formed by the lens) and to the properties R_1, R_2 and n of the lens. It is valid, of course, only for paraxial rays and only if the lens is very thin. For nonparaxial rays, and for nonthin lenses, the image may not be sharp.

If we consider an object at infinity $(d_0=\infty)$, the image distance is the focal length, $d_i=f$, and then the equation becomes

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right). \qquad (2)$$

This is called the **lens-maker's equation**. It relates the focal length of any lens to the radii of curvature of its two surfaces and its index of refraction. A radius of curvature is positive if a surface is convex to the incoming light, and is negative if concave. If a lens is turned around, so light comes from the opposite direction, R_1 *and* R_2 exchange roles in Eq.(2), and they also change sign so the value of f remains the same. Thus the position of the focal point F is the same on both sides of a lens.

Combining Eqs.(1) and (2) yields

$$\frac{1}{d_0}+\frac{1}{d_i}=\frac{1}{f}.$$

This is called the **lens equation**. It will be valid for both converging and diverging lenses and for all object and image positions if we use the following conventions.

1. The focal length is positive for converging lenses and negative for diverging lenses. A radius of curvature is positive when light strikes a convex surface and negative when it strikes a concave surface.

2. The object distance is positive if it is on the side of the lens from which the light is coming, otherwise it is negative.

3. The image distance is positive if it is on the opposite side of the lens from where the light is coming; if it is on the same side, d_i is negative. We can also say that the image distance is positive for a real image and negative for a virtual image.

4. Object and image heights, h_o and h_i, are positive for points above the axis, and negative for points below the axis.

The **lateral magnification m** of a lens is defined as the ratio of the image height to object height,

$$m = \frac{h_i}{h_0} = -\frac{d_i}{d_0}.$$

For an upright image the magnification is positive, and for an inverted image m is negative.

Problem 4-13. A planoconcave lens has one flat surface and the other has R = 18.4 cm. The index of refraction of the lens is n = 1.51. What is the focal length?

Solution: A plane surface has infinite radius of curvature (so $1/R_1 = 0$). Therefore we can write

$$\frac{1}{f} = (1.51 - 1)\left(-\frac{1}{18.4}\right)$$

and we have f = -36 cm, and the lens is diverging.

Problem 4-14. What is the position and size of the image of a 22.4 cm high object placed 1.5 m from a +50 mm focal length lens?

Solution: The lens is converging with f = +5 cm and the lens equation may be written as

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0} = \frac{29}{150} cm^{-1}$$

and so $d_i = 5.17\ cm$ behind the lens.
The magnification

$$m = -\frac{d_i}{d_o} = -\frac{5.17}{150} = -0.0345$$

and thus

$$h_i = (-0.0345)(22.4) = -0.773\ cm.$$

The image is 7.73 mm high and is inverted.

Problem 4-15. An object is placed 10 cm from a 15-cm focal length converging lens. Determine the image position and size.

Solution: Since f = 15 cm and $d_o = 10\ cm$,

$$\frac{1}{d_i} = -\frac{1}{30} cm^{-1}$$

so $d_i = -30\ cm$. Since d_i is negative, the image must be virtual and on the same side of the lens as the object.
The magnification m = -(-30)/10 = -3; so the image is three times as large as the object and is upright.

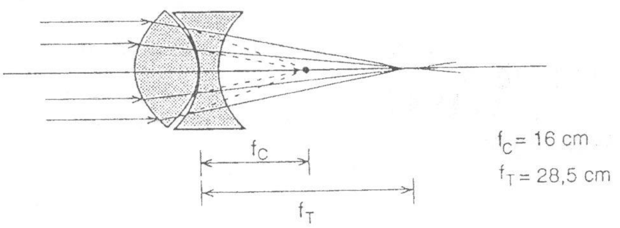
Problem 4-16. Where must a n object be placed if a 25-cm focal length diverging lens is to form a virtual image 20 cm in front of the lens/

Solution: Since f = -25 cm and $d_i = -20\ cm$, then

$$\frac{1}{d_0} = -\frac{1}{25} + \frac{1}{20} = \frac{1}{100} cm^{-1}.$$

So the object must be 100 cm in front of the lens.

Problem 4-17. To measure the focal length of a diverging lens, a converging lens is placed next to it as in Fig. The rays are focused by this combination at a point 28.5 cm behind them. If the converging lens has a focal length f_c of 16 cm, what is the focal length f_d of the diverging lens.



Solution: Let $f_T = 28.5\ cm$ refer to the focal length of the total combination. If the diverging lens were absent, the converging lens would form the image at its focal point, that is, at a distance $f_c = 16\ cm$ behind it. When the diverging lens is placed next to the converging lens we treat the image formed by the first lens as the object for the second diverging lens. Since this object lies to the right of the diverging lens, this is a situation where d_o is negative. Thus, for the diverging lens, the object is virtual and $d_0 = -16\ cm$; and it forms the image at distance $d_i = 28.5\ cm$ away. Thus

$$\frac{1}{f_d} = \frac{1}{d_0} + \frac{1}{d_i} = -\frac{1}{16} + \frac{1}{28.5} = -0.0274\ cm^{-1}.$$

So $f_d = -36.5\ cm$.

This last problem is an illustration of how to deal with lenses used in combination. In general, when light passes through several lenses, the image formed by one lens becomes the object for the next lens. The total magnification will be the product of the separate magnifications of each lens.

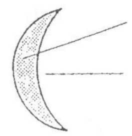
Problem 4-18. A convex meniscus lens is made from glass with n = 1.5. The radii of curvature are 22.4 cm and 46.2 cm. (a) What is the focal length? (b) Where will it focus an object 2 m away?

Solution: (a) $R_1 = 22.4\,cm$ and $R_2 = 46.2\,cm$; both are positive since both are convex surfaces to the incoming light assumed from the left. Then

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{22.4} - \frac{1}{46.2}\right) = 0.0114\,cm^{-1}.$$

So f = 89 cm and is converging.

Notice that if we turn the lens around so that $R_1 = -46.2\,cm$ and $R_2 = -22.4\,cm$ we get the same result.



(b) From the lens equation

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = 0.62\,m^{-1}$$

so $d_i = 1.6\,m$.

Problem 4-19. Both surfaces of a double convex lens have radii of 28 cm. If the focal length is 26.2 cm, what is the index of refraction of the lens material?

[1.53]

Problem 4-20. How far apart are an object and an image formed by a 65-cm focal length converging lens if the image is 3× larger than the object and is real?

[3.5 m]

Problem 4-21. Two 32-cm focal length converging lenses are placed 21.5 cm apart. An object is placed 55 cm in front of one. Where will the final image formed by the second lens be located? What is the total magnification?

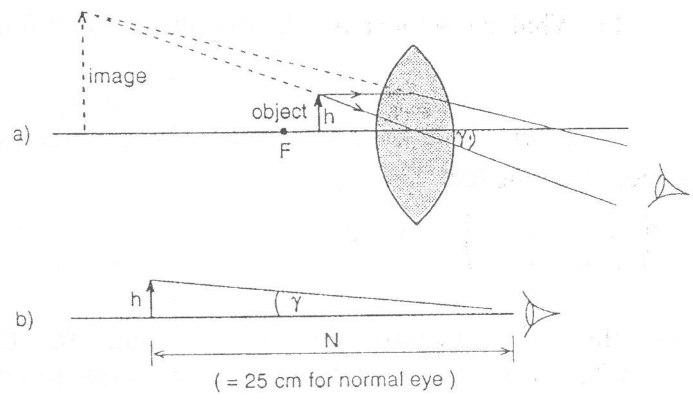
[20.2 cm beyond second lens; 0.512]

4.6 Optical Instruments

I) The Simple Magnifier

The nearest point at which an eye can focus clearly is called the **near point**. The standard near point is taken to be 25 cm. The most distant point that can be focused clearly by the eye is called the **far point**. For a normal eye the far point is taken to be infinity.

A simple magnifier allows to place a small object closer to the eye so that it subtends a greater angle. The object is placed at the focal point or near it. Then the converging lens produces a virtual image, which must be at least 25 cm from the eye if the eye is to focus on it. A comparison of Fig.(a) with (b), in which the same object is



viewed at the near point with the unaided eye, reveals that the angle the object subtends at the eye is much larger when the simple magnifier is used.

The **angular magnification, M,** of the lens is defined as the ratio of the angle subtended by the object using the lens to the angle subtended using the unaided eye with the object at the near point of the eye

$$M = \frac{\gamma'}{\gamma}.$$

This magnification can be written in terms of the focal length f of the lens as follows.

a) If the eye is focused at near point N = 25 cm (the image is at the near point N) then

$$M = 1 + \frac{N}{f}.$$

b) If the eye is relaxed (eye is focused at ∞) when using the simple magnifier, the image is then at infinity, and the object is then precisely at the focal point. Then

$$M = \frac{N}{f}.$$

It is clear that the magnification is slightly greater when the eye is focused at its near point than when relaxed. And the shorter the focal length of the lens, the greater the magnification.

Problem 4-22. An 8-cm focal length converging lens is used as a simple magnifier. Calculate: (a) the maximum magnifications; (b) the magnification when the eye is relaxed.

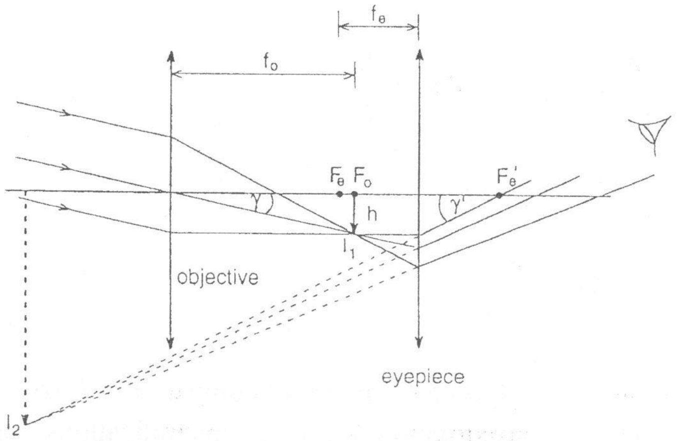
Solution: (a) The maximum magnification is obtained when the eye is focused at its near point:

$$M = 1 + \frac{N}{f} = 1 + \frac{25}{8} \approx 4\times.$$

b) With the eye focused at infinity,

$$M = \frac{25}{8} \approx 3\times.$$

II) Telescope

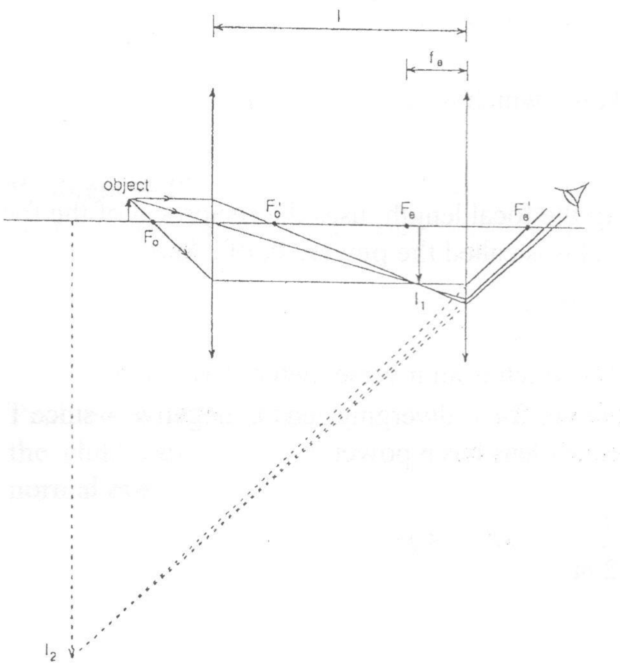


A telescope is used to magnify objects that are very far away; in most cases the object can be considered to be at infinity. The astronomical telescope contains two converging lenses located at opposite ends of a long tube. The lens closest to the object is called the **objective lens** and forms a real image I_1 of the object at its focal point F_0 (or near it if the object is not at infinity). Although this image, I_1, is smaller than the original object, it is very close to the second lens, called the **eyepiece**, which acts as a magnifier. The eyepiece magnifies the image produced by the objective to produce a second, greatly magnified image, I_2, which is virtual. If the viewing eye is relaxed, the eyepiece is adjusted so the image I_2 is at infinity. Then the real image I_1 is at the focal point F_e of the eyepiece, and the distance between the lenses is $f_0 + f_e$ for an object at infinity.

The **magnification** of this telescope is

$$M = -\frac{f_0}{f_e},$$

where a minus sign indicates that the image is inverted. To achieve a large magnification, the objective lens should have a long focal length and the eyepiece a short one.



Problem 4-23. A telescope has an objective lens whose focal length is 28 cm and an eyepiece with focal length -8 cm. What is its magnification?

Answer:

$M = -f_0 / f_e = -(28)(-8) = 3.5\times$.

III) Compound Microscope

A compound microscope has both objective and eyepiece lenses. A microscope is used to view objects that are very close, so the object distance is very small. The object is placed just beyond the objective's focal point. The image I_1 formed by the objective lens is real, quite far from the lens, and

much enlarged. This image is magnified by the eyepiece into a large virtual image I_2, seen by the eye.
The magnification M is equal to the product of the objective magnification M_0 and the eyepiece magnification M_e;

$$M_0 = \frac{d_i}{d_o} = \frac{l - f_e}{d_0} \quad ; \quad M_e = \frac{N}{f_e} \; ;$$

and

$$\mathrm{M} = M_0 \times M_e \; .$$

This can be expressed as

$$M \approx \frac{N\,l}{f_e\,f_o} \; .$$

This approximation is accurate when f_e *and* f_0 are small compared to l so $l - f_e \approx l$ *and* $d_0 \approx f_0$. This is a good approximation for large magnifications, since these are obtained when f_e *and* f_0 are very small.

Problem 4-24. A compound microscope consists of a 10× eyepiece and a 50× objective 18 cm apart.
Determine: (a) the magnification ; (b) the focal length of each lens; and (c) the position of the object when the final image is in focus with the eye relaxed. Assume a normal eye with N = 25 cm.

Solution: (a) The magnification is 10×50 = 500× .

(b) The eyepiece focal length is $f_e = N / M_e = 25/10 = 2.5\,cm$. It is easier to next find d_0 before we find f_0 since we can use the equation for M_0 ; solving for d_0 , we find $d_0 = (l - f_e)/M_0 = 15.5/50 = 0.31\,cm$. Then, from the lens equation:

$$\frac{1}{f_0} = \frac{1}{d_0} + \frac{1}{d_i} = \frac{51}{15.5}\,cm^{-1} \; ;$$

so $f_0 = 0.3\,cm$.

(c) We just calculated $d_0 = 0.31\,cm$, which is very close to f_0 .

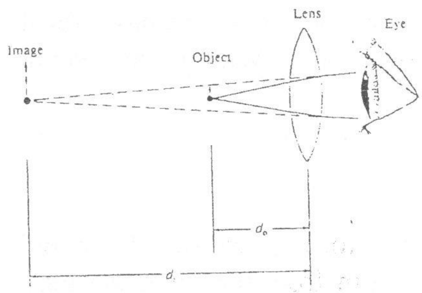
IV) Ophthalmology, instead of using the focal length, uses the reciprocal of the focal length to specify the strength of lenses. This is called the **power, P,** of a lens:

$$P = \frac{1}{f}$$

The unit for lens power is the diopter (D) which is an inverse meter: 1 D = 1 m^{-1} .
The power of a converging lens is positive, for a diverging lens is negative - since f is negative. For example, a 20-cm focal length lens has a power

$$P = \frac{1}{0.2\,m} = 5\,m^{-1} = 5\,D \, .$$

Problem 4-25. A farsighted person has a near point of 100 cm. Reading glasses must have what lens power so that this person can read at a distance of 25 cm? Assume the lens is very close to the eye.



Solution: When the object is placed 25 cm from the lens, we want the image to be 100 away on the same side of the lens, and so it will be virtual. Thus $d_0 = 25\,cm$, $d_i = -100\,cm$ and

$$\frac{1}{f} = \frac{1}{25} + \frac{1}{-100} = \frac{1}{33}\,cm^{-1}.$$

So f = 33 cm = 0.33 m.

And the power of the lens is P = 1/f = +3 D. The plus sign indicates it is a converging lens.

Problem 4-26. A nearsighted eye has near and far points of 12 cm and 17 cm, respectively. What lens power is needed for this person to see distant objects clearly, and what then will be the near point? Assume that each lens is 2 cm from the eye.

Solution: The lens must image distant objects $(d_0 = \infty)$ so they are 17 cm from the eye, or 15 cm in front of the lens $(d_i = -15\,cm)$:

$$\frac{1}{f} = \frac{1}{-15\,cm}.$$

So f = - 15 cm = - 0.15 m and P = 1/f = -6.7 D. The minus sign indicates it must be a diverging lens. For the near point, the image must be 12 cm from the eye or 10 cm from the lens, so $d_i = -0.1\,m$ and

$$\frac{1}{d_0} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{-0.15} - \frac{1}{-0.10} = \frac{1}{0.30}\,m^{-1}.$$

So $d_0 = 30\,cm = 0.30\,m$ which means the near point when the person is wearing glasses is 30 cm in front of the lens.

Problem 4-27. A magnifier is rated at 3× for a normal eye focusing on an image at its near point. (a) What is its focal length? (b) What will it be if the 3× referred to a relaxed eye?

[12.5 cm; 8.3 cm]

Problem 4-28. A child has a near point of 10 cm. What is the maximum magnification the child can obtain using an 8.8-cm focal length magnifier? Compare to that for a normal eye.

[2.1× for child; 3.8× for normal eye]

Problem 4-29. A small object is placed 3.8 cm from a +4-cm focal length lens. Calculate (a) the position of the image, (b) the angular magnification.

[76 cm in front of lens; 6.6×]

Problem 4-30. What is the magnification of a telescope whose objective lens has a focal length of 50 cm and whose eyepiece has a focal length of 3.1 cm? What is the overall length of the telescope when adjusted for a relaxed eye?

[M = -16; 53.1 cm]

Problem 4-31. The eyepiece of a compound microscope has focal length of 2.5 cm and the objective has f = 0.8 cm. If an object is placed 0.85 cm from the objective lens, calculate (a) the distance between the lenses when the microscope is adjusted for a relaxed eye, and (b) the total magnification.

[16.1 cm; 160×]

Problem 4-32. A microscope has a 2-cm focal length eyepiece and 1-cm focal length objective. Calculate (a) the position of the object if the distance between the lenses is 18 cm, and (b) the total magnification, assuming a relaxed normal eye.

$[d_0 = 1.07\, cm;\ 190\times]$

Problem 4-33. Reading glasses of what power are needed for a person whose near point is 120 cm so that he can read at 25 cm? Assume a lens-eye distance of 2 cm.

[+3.5 D]