

$$\left[W = Q \frac{T_1 - T_2}{T_1} = 1.48 \times 10^5 \text{ J} \right]$$

Problem 2-51. A Carnot refrigerator absorbs heat from the freezer compartment so that it changes water of temperature $t_1 = 20^\circ\text{C}$ and mass $m = 3.6 \text{ kg}$ into ice of temperature $t_2 = 0^\circ\text{C}$ in time $\tau = 1 \text{ hour}$. If the compressor output is 100 W calculate the heat which must be exhausted into the room. The specific heat of water is $c = 4186 \text{ J/(kg.K)}$, the heat of fusion is $l = 334 \text{ kJ/kg}$.

$$\left[Q_1' = P'\tau + m(c \Delta t + l) = 1.87 \times 10^6 \text{ J} \right]$$

2.4 CHANGES OF PHASE, REAL GASES

When a material changes phase from a solid to a liquid, or from a liquid to a gas, a certain amount of energy is involved in this **change of phase**.

The heat required to change one kilogram of a substance from the solid to the liquid state is called **heat of fusion** l_F .

The heat required to change one kilogram of a substance from the liquid to the vapour is called **heat of vaporisation** l_v .

Heats of fusion and vaporisation are also called **latent heats**.

Heats of vaporisation and fusion also refer to the amount of heat released by a substance when it changes from a gas to a liquid, or from a liquid to a solid.

Clausius-Clapeyron's equation expresses the dependence of the latent heat of the substance on the change in its temperature and pressure:

$$l = \frac{T(V_2 - V_1)}{m} \frac{dP}{dT}$$

where V_2 and V_1 are the volume of the substance in the higher and lower phase and m is the mass of the substance.

The behaviour of real gases at high pressures and near their liquefaction point deviates from the ideal gas law. These deviations are due to the finite size of molecules and to the attractive forces between molecules which become greater in these situations. An equation which describes the behaviour of real gases is **van der Waal's equation of state**:

$$\left(P + \frac{n^2 a}{V^2} \right) (V - nb) = nRT$$

where constants a and b are experimental constants that are different for different gases.

Problem 2-52 If 1 kg of ice at temperature -20°C is heated at atmospheric pressure until all the ice has been changed into steam, how much heat is required? Specific heat of ice $c_{ice} = 2.05 \text{ kJ/kg.K}$, specific heat of water is $c_w = 4.18 \text{ kJ/kg.K}$, latent heat of fusion is $l_F = 334 \text{ kJ/kg}$ and latent heat of vaporisation is $l_v = 2257 \text{ kJ/kg}$.

Solution: We first determine the heat needed to raise the temperature of ice from -20°C to 0°C :

$$Q_1 = m \cdot c_{ice} \cdot \Delta T = 1 \times 2.05 \times 20 = 41 \text{ kJ}$$

The heat needed to melt one kg of ice is

$$Q_2 = m \cdot l_F = 1 \times 334 = 334 \text{ kJ.}$$

The heat needed to raise the temperature of the resulting 1 kg of water from 0 to 100°C is

$$Q_3 = m \cdot c_w \cdot \Delta T = 1 \times 4.18 \times 100 = 418 \text{ kJ}$$

where we have disregarded any variation of specific heat of water over this range of temperatures. Finally the heat needed to vaporise 1 kg of water at 100°C is

$$Q_4 = m \cdot l_v = 2260 \text{ kJ.}$$

The total amount of heat required is therefore:

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = 3.05 \text{ MJ.}$$

Problem 2-53. The fusion temperature of lead at pressure $P_1 = 1 \times 10^5 \text{ Pa}$ is 327°C . Calculate the heat of fusion l_F of lead at this temperature if due to the increase of external pressure by $\Delta P = 1 \times 10^5 \text{ Pa}$ the temperature increases by $\Delta t = 0.008^{\circ}\text{C}$ and lead increases on melting its volume by $k = 3.4\%$. The density of lead is $\rho = 11.3 \times 10^3 \text{ kg.m}^{-3}$.

Solution: We determine the heat of fusion from Clausius-Clapeyron's equation:

$$l_F = \frac{T(V_2 - V_1)}{m} \frac{dP}{dT}$$

We can calculate the volume of the liquid phase V_2 from the condition that its volume is about 3.4% greater than the volume of solid phase V_1 :

$$V_2 = V_1 + \frac{V_1}{100} \cdot k$$

Since ΔT is very small we can substitute in Clausius-Clapeyron's equation the first derivative $\frac{dP}{dT}$ by the term $\frac{\Delta P}{\Delta T}$. Thus we have

$$l_F = \frac{T \left(V_1 + \frac{V_1}{100} k - V_1 \right)}{m} \frac{\Delta P}{\Delta T} = \frac{T \cdot V_1}{m} \frac{k}{100} \frac{\Delta P}{\Delta T}$$

For solid state we know that $\frac{V_1}{m} = \frac{1}{\rho}$ where ρ is the density of lead. Substituting numerical values we finally obtain

$$I_F = \frac{T}{\rho} \frac{k}{100} \frac{\Delta P}{\Delta T} = \frac{600}{11.3 \times 10^3} \frac{3.4}{100} \frac{1 \times 10^5}{0.008} = 22.5 \times 10^3 \text{ J.kg}^{-1}.$$

Problem 2-54 Oxygen of mass $m=4 \text{ kg}$ is enclosed at temperature $t=29^\circ\text{C}$ in a container of volume $V=2 \text{ m}^3$. (a) Calculate its pressure. (b) If the oxygen is heated at constant volume so that its temperature is doubled, determine its pressure by using van der Waals' equation. ($a = 1.37 \times 10^5 \text{ J.m}^3.\text{kmol}^{-2}$; $b = 0.03 \text{ m}^3.\text{kmol}^{-1}$; $M_{O_2} = 32 \text{ kg.kmol}^{-1}$).

Solution: From van der Waals' equation we express pressure as

$$P = \frac{n.R.T}{V - n.b} - n^2 \frac{a}{V^2}$$

Substituting numerical values we have for pressure of oxygen before isochoric process:

$$P = \frac{\frac{4}{32} \cdot 8314 \times 302}{2 - \frac{4}{32} \times 0.03} - \left(\frac{4}{32}\right)^2 \frac{1.37 \times 10^5}{2^2} = 1.57 \times 10^5 \text{ Pa}$$

The change of pressure after isochoric process is

$$\Delta P = P' - P = \frac{nR(T_0 + 2t)}{V - nb} - n^2 \frac{a}{V^2} - \frac{n.R.(T_0 + t)}{V - nb} + n^2 \frac{a}{V^2} = \frac{n.R.t}{V - nb}$$

where $T_0 = 273.15 \text{ K}$.

Finally for the change of pressure we have

$$\Delta P = \frac{\frac{4}{32} \cdot 8314 \times 29}{2 - \frac{4}{32} \cdot 0.03} = 1.509 \times 10^4 \text{ Pa}.$$

Problem 2-55. Calculate the heat of vaporisation of water at temperature 100°C and atmospheric pressure if 1 kg of water at this temperature has volume $V_1 = 10^{-3} \text{ m}^3$, the volume of saturated vapour at the same temperature is $V_2 = 1.647 \text{ m}^3$ and the change in pressure of saturated vapour as a function of temperature is $\frac{dP}{dT} = 3604 \text{ Pa.K}^{-1}$.

$$[I_v = 2.25 \times 10^6 \text{ J.kg}^{-1}]$$

Problem 2-56 Water of mass $m=0.5 \text{ kg}$ changes completely into steam of volume 0.837 m^3 at temperature 100°C and at atmospheric pressure. Calculate the work done by the steam and the increase of its internal energy. The heat of vaporisation of water is $I_v = 2.25 \times 10^6 \text{ J.kg}^{-1}$.

$$\left[A = \int_1^2 P dV = 0.837 \times 10^5 J \right]$$

$$\left[\Delta U = Q - A = 1.041 \times 10^6 J \right]$$

2.5 HEAT TRANSFER

Heat is transferred from one body to another, or from one point to another within a body, by three different processes.

1. Conduction - the heat diffuses through a solid material or a stagnant fluid.
2. Convection - a moving liquid or gas absorbs heat at one place and gives it up at another place.
3. Radiation - the transfer of heat from one place to another is by means of electromagnetic waves.

The heat that flows due to conduction through a slab of material from its hot face to its cold face is

$$Q = \lambda S \frac{T_{hot} - T_{cold}}{l} t$$

where λ is the thermal conductivity of the material, S is the cross sectional area of the slab and l is its thickness.

The heat transferred from a fluid to a wall in time t is

$$Q = \alpha S t (T_f - T_w)$$

where α stands for heat-transfer coefficient, T_f and T_w stand for the temperatures of the fluid and wall respectively and S stands for the cross sectional area.

The rate at which an object radiates energy is proportional to the area S of the emitting object and to the fourth power of the thermodynamic temperature:

$$\frac{\Delta Q}{\Delta t} = e \sigma S T^4$$

In this expression e is the emissivity of the material and σ is Stefan-Boltzmann's constant.

Problem 2-57. A major source of a heat loss from a house occurs through the windows. Calculate the rate of heat flow through a glass window $2.0 \times 1.5 \text{ m}$ in area and 3.2 mm thick if the temperatures at the inner and outer surfaces are 15.0°C and 14.0°C respectively. The thermal conductivity of glass is $\lambda = 0.84 \text{ J/s.m}$.

Solution: The rate of heat flow is defined as $\frac{\Delta Q}{\Delta t}$. Using the expression for heat flow we obtain

$$\frac{\Delta Q}{\Delta t} = \lambda S \frac{T_{hot} - T_{cold}}{l} = 0.84 \times (2 \times 1.5) \times \frac{15 - 14}{3.2 \times 10^{-3}} = 790 \text{ W}$$

Problem 2-58. Consider a compound slab, consisting of two materials having different thicknesses l_1 and l_2 and different thermal conductivities λ_1 and λ_2 . If the temperatures of the outer surfaces are T_2 and T_1 find the rate of heat transfer through the compound slab in a steady state.

Solution: Let T_x be the temperature at the interface between the two materials. Then we can write for the rate of heat transfer through the second slab

$$\frac{\Delta Q_2}{\Delta t} = \lambda_2 S \frac{T_2 - T_x}{l_2}$$

Similarly we have for the first slab

$$\frac{\Delta Q_1}{\Delta t} = \lambda_1 S \frac{T_x - T_1}{l_1}$$

In a steady state $\frac{\Delta Q_2}{\Delta t} = \frac{\Delta Q_1}{\Delta t} = \frac{\Delta Q}{\Delta t}$ which is the rate of heat transfer for both sections. Thus we have

$$\lambda_2 S \frac{T_2 - T_x}{l_2} = \lambda_1 S \frac{T_x - T_1}{l_1}$$

Solving for T_x and substituting into either of these equations we obtain

$$\frac{\Delta Q}{\Delta t} = \frac{S(T_2 - T_1)}{\left(\frac{l_1}{\lambda_1}\right) + \left(\frac{l_2}{\lambda_2}\right)}$$

Problem 2-59. A ceramic teapot ($e=0.70$) and a shiny one ($e=0.10$) each hold 0.75 l of water at 95 °C. Estimate the rate of heat loss from each.

Solution: A teapot that holds 0.75 l can be approximated by a cube with sides 10 cm in length, so its surface area would be about $5 \times 10^{-2} \text{ m}^2$. The rate of heat loss would be

$$\frac{\Delta Q}{\Delta t} = e\sigma S(T_1^4 - T_2^4)$$

Substituting numerical values we obtain for the ceramic pot $\frac{\Delta Q}{\Delta t} = 20 \text{ W}$ and for the

shiny one $\frac{\Delta Q}{\Delta t} = 3 \text{ W}$.