

4 GEOMETRICAL OPTICS

The ratio of the speed c_0 of light in vacuum to its speed c in a given material is called the **index of refraction** n of that material:

$$n = \frac{c_0}{c}$$

The accepted value today for the speed of light in vacuum is $2.99792458 \times 10^8 \text{ m s}^{-1}$. Usually we round this off to $3 \times 10^8 \text{ m s}^{-1}$ when extremely precise results are not required. In air the speed is only slightly less. In other transparent materials such as glass and water, the speed of light is always less than that in vacuum.

4.1 Reflection of Light

When light strikes the surface of an object some of light is reflected. The rest is either absorbed by the object and transformed to heat or, if the object is transparent like glass or water part of it is transmitted through.

We define the **angle of incidence** α_i to be the angle an incident ray makes with the normal to the surface and the **angle of reflection** α_r to be the angle the reflected ray makes with the normal.

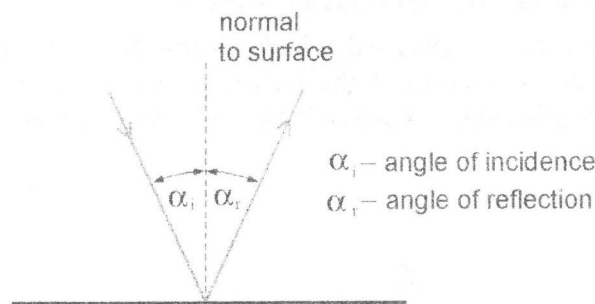


Figure 4-1

For flat surface the incident and reflected rays lie in the same plane with the normal to the surface and the angle of incidence equals the angle of reflection. This is the **law of reflection**.

4.2 Formation of Image by Plane Mirror

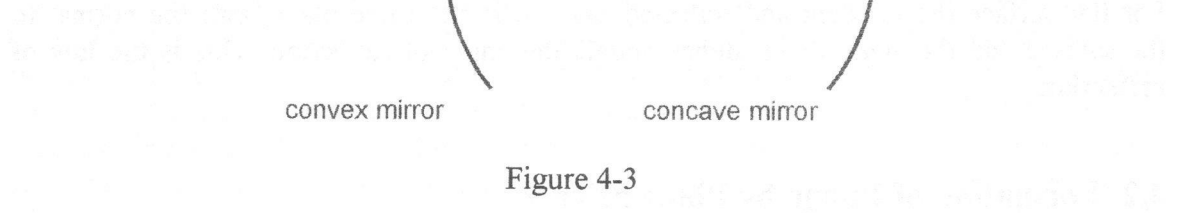
In Fig.4-2 it is shown how an image is formed by a plane mirror. Rays leave each point on the object going in many directions.

The diverging rays that enter the eye appear to come from behind the mirror as shown by the dashed lines. The point from which the ray seems to come is one point on the image. For each point on the object there is a corresponding image point.

Let us assume the ray that leaves the point A on the object and strikes the mirror at point B. The angles ABD and CBD are equal because of the law of reflection and hence, $AD=CD$ and the image distance d_i equals the object distance d_o and the height of the image is the same as that of the object.

Since the rays do not pass through the image, a film placed at the image would not detect the image. Therefore, it is called a **virtual image**. As for a real image, the light passes

The most common mirrors are spherical which means they form a section of a sphere. A spherical mirror is called **convex** if the reflecting surface is on the outer surface of the spherical shape and it is called **concave** if the reflecting surface is on the inner surface of the sphere.



To see how the spherical mirror forms an image let us consider the rays which are parallel to the principal axis and falling on a concave mirror. The **principal axis** is defined as the straight line perpendicular to the curved surface at its centre (Fig.4-4). The law of reflection holds for each point when the rays parallel to the principal axis strike the mirror. After reflection such rays will cross each other at a single point which is called the focal point F of the mirror. **Focal point** F is the point in which the rays parallel to the principal axis come to a focus. We can also say that the focal point is the image point for an object infinitely far away along the principal axis. The distance between F and the centre of the mirror, length FA , is called the **focal length** f of the mirror.

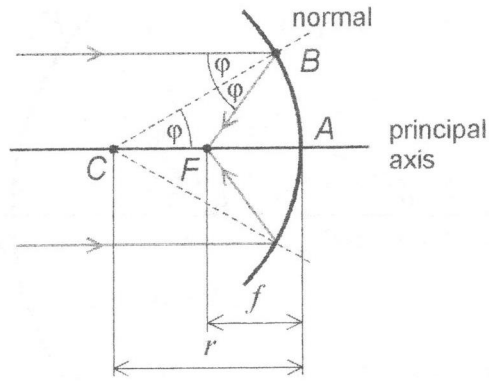


Figure 4.4

How do we calculate the focal length f of the mirror? Let us consider a ray that strikes the mirror at B. The point C is the centre of curvature of the mirror (the centre of the sphere of which the mirror is a part). So, the dashed line CB is equal to r – the radius of mirror and CB is also normal to the surface at B. Since the angles FCB and CBF must be equal, the length CF must be equal BF. We assume the mirror is small compared to its radius of curvature, so the angles are small and the length FB is nearly equal to length FA. In this approximation, $FA=FC$. But $FA=f$ and $CA=2FA=r$. Thus the focal length is half

the radius of curvature
$$f = \frac{r}{2} \quad (4-1)$$

We can say that for an object at infinity the image is located at the focal point of a concave spherical mirror and its position is given by Eq.4-1.

To find the image for an object OO' not at infinity we use three particularly simple rays (see Fig.4-5):

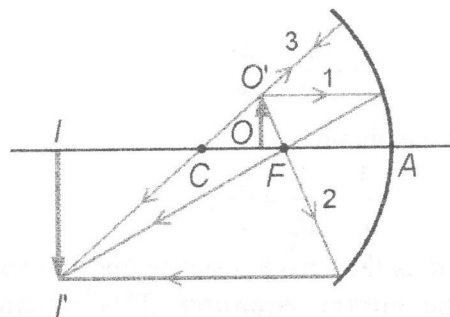


Figure 4-5

Ray 1 is going parallel to the axis and therefore it must pass through F.

Ray 2 is passing through F and therefore it must reflect to be parallel to the axis.

Ray 3 is going along a radius of the curvature of the spherical surface and therefore it is perpendicular to the mirror and will be reflected back.

The point at which these rays cross is the image I' .

Let us calculate the position of the image for a given object.

In Fig.4-6 we assume an arbitrary ray, say OP, leaving a point O on the object. It reflects from the mirror and passes through point I which will be its image point. We can write:

$$\gamma + \delta = 180^\circ,$$

and for triangle OIP

$$\alpha + 2\phi + \delta = 180^\circ = \gamma + \delta.$$

Thus we have

$$\alpha + 2\phi = \gamma. \quad (a)$$

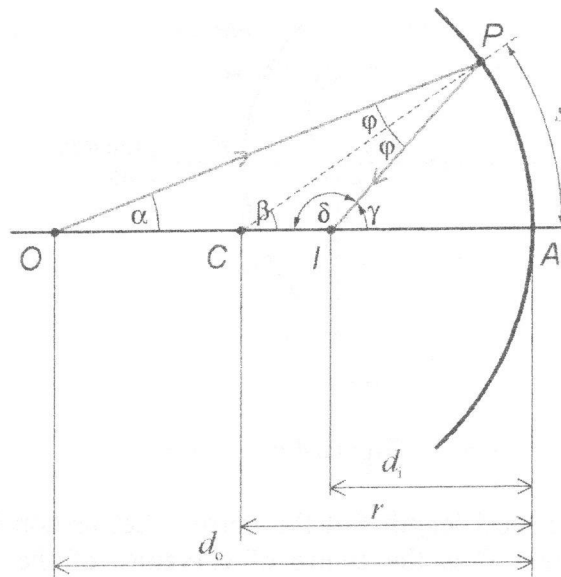


Figure 4-6

For triangle OCP we can write:

$$\alpha + \varphi + (180^\circ - \beta) = 180^\circ,$$

or

$$\alpha + \varphi = \beta. \quad (b)$$

From relations (a), (b) we eliminate φ and obtain

$$\alpha + \gamma = 2\beta.$$

If α, β, γ are small angles, this relation can be written (in radian measure) to be a good approximation as

$$\frac{s}{d_o} + \frac{s}{d_i} = \frac{2s}{r}.$$

If we divide by s and use Eq.4-1 we obtain

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}, \quad (4-2)$$

where d_o is the object distance, d_i is the image distance and r is the radius of curvature of the mirror. Eq.4-2 is called the **mirror equation**. This equation gives us a way of determining the position of the image if the object position and the focal length (or radius of curvature) of the mirror are given.

The **lateral magnification** m of a mirror is defined as the height of the image h_i divided by the height h_o of the object:

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}, \quad (4-3)$$

where we take the following conventions: if h_o is considered positive, h_i is positive if the image is upright and negative if inverted; d_i and d_o are positive if image and object are on the reflecting side of the mirror. So, the magnification is positive for an upright image and negative for an inverted image.

Example: 1.5 cm high object is placed 20 cm from a concave mirror whose radius of curvature is 30 cm. Calculate the position of the image and its size.

Solution: The focal length $f = \frac{r}{2} = 15 \text{ cm}$. From the mirror equation we have

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{15} - \frac{1}{20} = 0.0167 \text{ cm}^{-1}.$$

So, $d_i = 60 \text{ cm}$ and the image is 60 cm from the mirror on the same side as the object.

The lateral magnification is $m = -\frac{60}{20} = -3$, so, the height of the image $h_i = m \times h_o = (-3) \times (1.5) = -4.5 \text{ cm}$ and the image is inverted.

The analysis used for concave mirror can be applied for **convex mirror**. The parallel rays falling on a convex mirror diverge after reflecting and seem to come from point F behind the mirror. This is called the focal point and its distance from the centre of the mirror represents the focal length f which is again equal $f = \frac{r}{2}$ (r is the radius of curvature of the mirror).

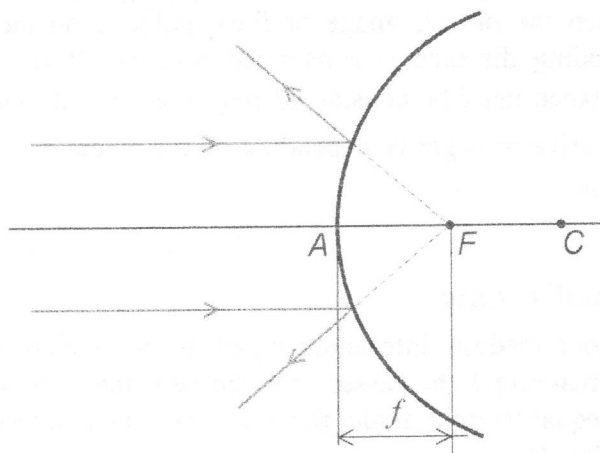


Figure 4-7

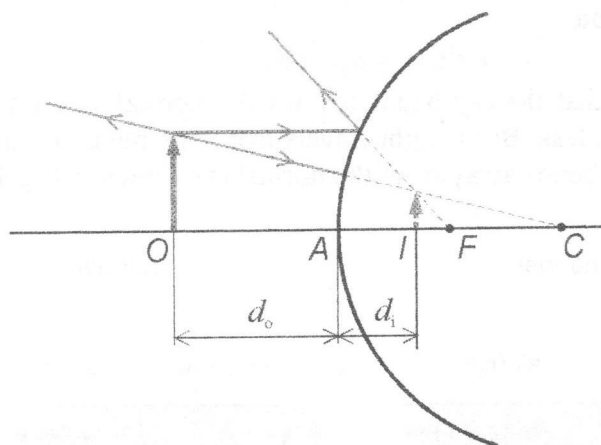


Figure 4-8

No matter where the object is placed on the reflecting side of the mirror, the image will be virtual and erect (see Fig. 4-8).

The mirror equation Eq.(4-2) and the relation for the magnification Eq.(4-3) are also valid for convex mirror but the focal length (or the radius of curvature) must be considered negative.

Example: A convex car mirror has a radius of curvature of 40 cm. Calculate the location of the image and its magnification for an object 10 m from the mirror.

Solution: Since we have a convex mirror, its radius of curvature is negative, $r = -40$ cm, and its focal length $f = -20$ cm.

From the mirror equation we have

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-0.2} - \frac{1}{10} = -\frac{51}{10} \text{ m}^{-1}$$

So, $d_i = -\frac{10}{51} = -0.196$ m. The image is virtual one and is located behind the mirror.

The lateral magnification is $m = -\frac{d_i}{d_o} = -\frac{-0.196}{10} = 0.0196$, so the image is upright and is reduced by a factor of 51.

We summarise the rules as the **sign conventions** for applying Eqs.4-2 and 4-3 to concave and convex mirrors: when the object, image or focal point is on the reflecting side of the mirror the corresponding distance is considered positive. If any of these points is behind the mirror its distance must be considered negative. Object and image heights h_o and h_i are considered positive or negative depending on the object or image point is above or below the principal axis.

4.4 Refraction - Snell's Law

When light passes from one medium into another part of the incident light is reflected at the boundary and the remaining light passes into the new medium. If a ray of light is incident at an angle not equal to right angle, the ray is bent as it enters the new medium. This bending is called **refraction**.

The angle of refraction depends on the speed of light in the two media and on the incident angle. Relation between the incident angle and the angle of refraction is known as **Snell's law** or the **law of refraction**

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 \quad (4-4)$$

From this law it is clear that the ray bends toward the normal when entering a medium where the speed of light is less. But if light travels from one medium into a second where its speed is greater the ray bends away from the normal (as shown in Fig.4-9)

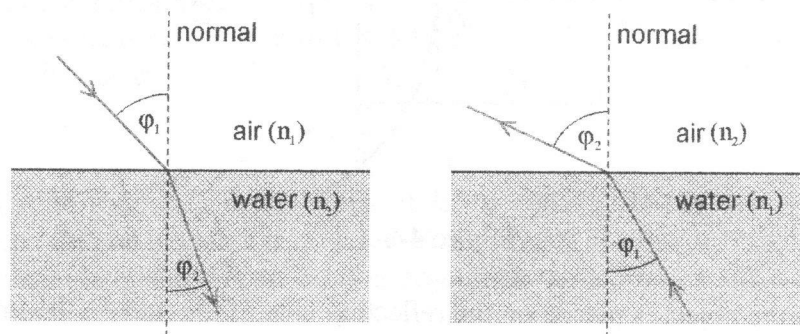


Figure 4-9

We can also say that if light enters a medium whose index of refraction n is greater (and its speed less) the ray is bent toward the normal (the angle of refraction is less than incident angle) but if light enters a medium whose n is less (and its speed greater) the ray bends away from the normal (the angle of refraction is greater than incident angle).

When light passes from one material into a second material where the index of refraction is less (the speed of light is greater) the light bends away from the normal. At a particular incident angle the angle of refraction will be 90° . Such the incident angle is called the **critical angle** φ_c . From Snell's law the angle φ_c is given by

$$n_1 \sin \varphi_c = n_2 \sin 90^\circ$$

or

$$\sin \varphi_c = \frac{n_2}{n_1} \quad (4-5)$$

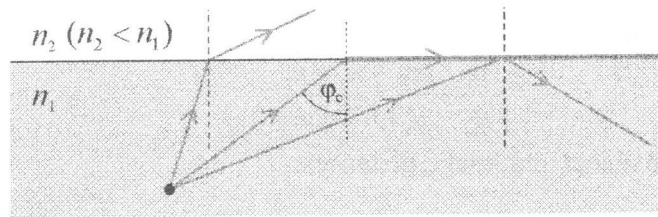


Figure 4-10

For incident angle greater φ_c Snell's law would tell us that $\sin \varphi_2$ is greater than 1. Thus in this case there is no refracted ray at all and all of the light is reflected. This is called **total reflection**.

4.5 Refraction at Spherical Surface

In this chapter we will examine the refraction of rays at the spherical surface of a transparent material.

Let n_1 be the index of refraction of a medium in which an object is located, let n_2 be the index of refraction of a transparent material ($n_2 > n_1$). The radius of curvature of the spherical boundary is R and its centre of curvature is at point C .

We will show that rays leaving a point O on the object will be focused at a single point I (called the image point) if we consider only rays that make a small angle with the axis; such rays are called **paraxial rays**.

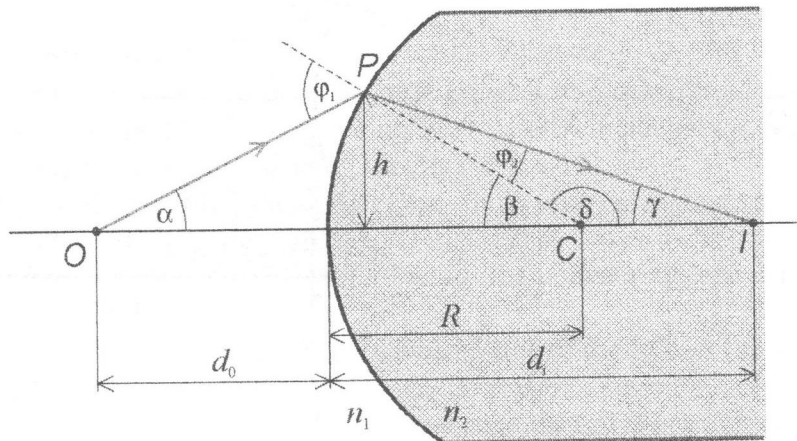


Figure 4-11

To do so, we consider a single ray leaving point O and striking the surface at point P (Fig.4-11). From Snell's law we can write

$$n_1 \sin \phi_1 = n_2 \sin \phi_2 .$$

Assuming all angles $\phi_1, \phi_2, \alpha, \beta$ to be small, the Snell's law holds the form

$$n_1 \phi_1 = n_2 \phi_2 .$$

For triangles CPI and COP we can write

$$\beta = \phi_2 + \gamma ,$$

$$\phi_1 = \alpha + \beta .$$

Since we consider only paraxial rays, we can use the approximations

$$\alpha = \frac{h}{d_o} , \quad \beta = \frac{h}{R} , \quad \gamma = \frac{h}{d_i} ,$$

and the Snell's law yields

$$\frac{n_1}{d_o} + \frac{n_2}{d_i} = \frac{n_2 - n_1}{R} , \quad (4-6)$$

where d_o and d_i are the object and image distances.

Eq.4-6 is called **paraxial equation** for the spherical refracting surface. It is clear that the image distance d_i , for a given distance d_o , does not depend on the angle of a ray. Hence all paraxial rays meet at the same point I called the image. Eq.4-6 allows us to calculate the image distance when the object distance and optical parameters (R, n_1, n_2) are given.

Eq.4-6 was derived for the convex spherical surface. If we make the following conventions it is also valid for the concave surface:

- if the surface is convex (the centre of curvature C is on the side of the surface opposite to that from which light comes) R is positive; for the concave surface (the centre C is on the same side from which the light comes) R is negative;
- the image distance is positive if the image is located on the opposite side from where light comes and negative if on the same side;
- the object distance is positive if the object is located on the same side from which light comes and negative if on the opposite side. By convention given above, for a concave surface (Fig.4-12) both R and d_i are negative; the image is virtual.

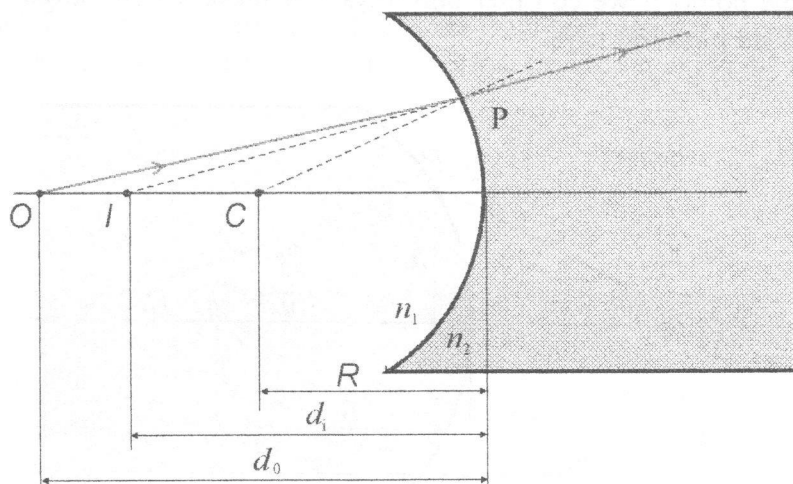


Figure 4-12

Example: We look down into 4 m lake. How deep does the lake appear to be? (Fig.4-13)

Solution: The rays come from the object point O on the lake bottom diverge after refraction and appear to come from the image point I.

If we put $d_o = 4$ m and for a flat surface $R = \infty$, we can use the paraxial equation (4-6).

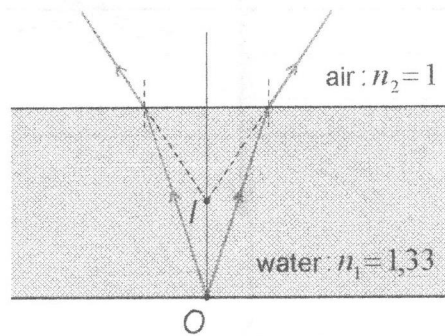


Figure 4-13

Substituting gives equation

$$\frac{1.33}{4} + \frac{1}{d_i} = 0,$$

and hence

$$d_i = -\frac{4}{1.33} = -3 \text{ m}.$$

The minus sign tells us the image point I is on the same side of the surface as the object point and thus the image is virtual. The lake appears to be 3 m deep only.

4.6 Thin Lenses

We assume the lens to be made of glass or transparent material so its index of refraction is greater than that of the air outside. If the diameter of the lens is small compared to the radii of curvature of the two lens surface, such the lens is called a **thin lens**. We distinguish two type of lenses.

Convex or converging lens:

Rays parallel to the axis converge to a point called the **focal point F** or we can also say that the focal point is the image point for an object on the axis at infinity. The distance from F to the lens is the **focal length f** (Fig.4-14).

Concave or diverging lens:

This lens makes parallel rays diverge. The **focal point F** of a diverging lens is defined as the point from which refracted rays originating from parallel incident rays seem to emerge. The distance from F to the lens is the **focal length f** (Fig.4-15).

For finding the image by ray tracing we use three particular rays as indicated for converging lens (Fig.4-16) and diverging lens (Fig.4-17).

Ray 1, is parallel to the axis and after refraction by the converging lens it passes through the focal point F behind the lens.

Ray 2, passes through the focal point F' on the same side of the lens as the object and after refraction by the lens is parallel to the axis.

Ray 3, is directed toward the centre of the lens and such the ray is not refracted by the lens.

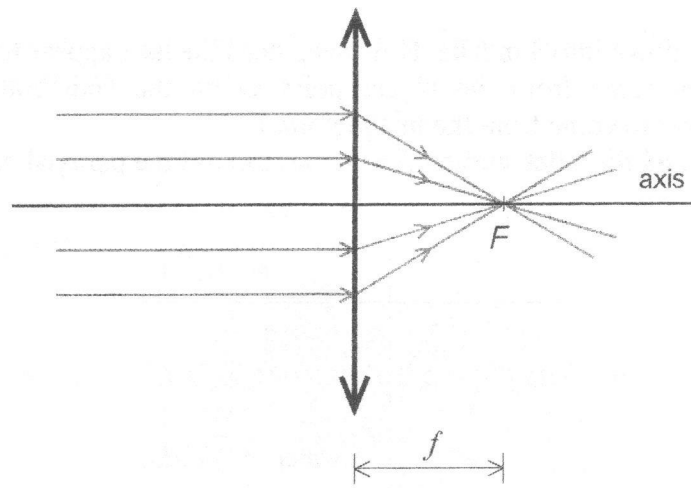


Figure 4-14

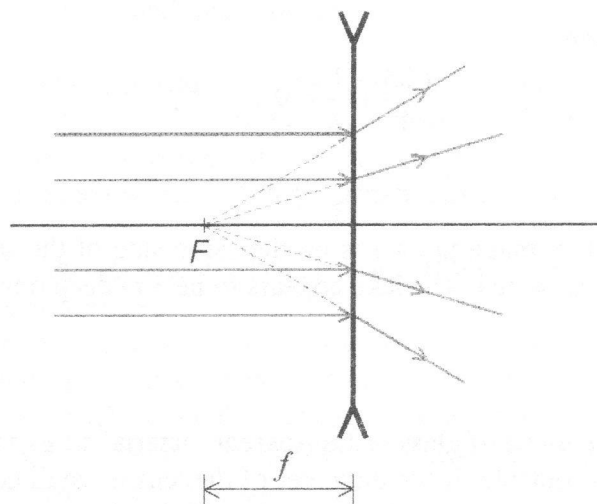


Figure 4-15

In this way we can find the image point for any point on the object and to determine the complete image of the object. Because the rays actually pass through the image for the case in Fig. 4-16, it is a **real image**.

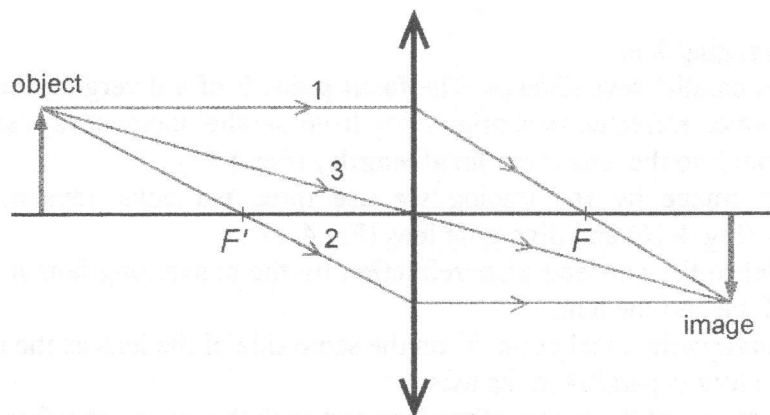


Figure 4-16

By using the same three rays we can determine the image for a diverging lens as shown in Fig.4-17.

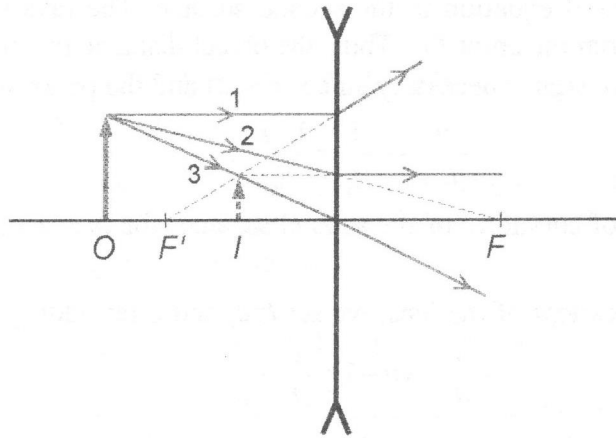


Figure 4-17

Ray 1, which is parallel to the axis after refraction seems to come from the focal point F' in front of the lens.

Ray 2, which is directed toward the focal point F behind the lens, is refracted parallel.

Ray 3, which is directed toward the centre of the lens is not refracted.

The three refracted rays seem to emerge from a point on the left of the lens that is called the image. Since the rays do not pass through the image, it is a **virtual image**.

4.7 Lens Equation

We derive an equation that relates the image distance to the object distance. Let us consider the lens in Fig.4-18 whose thickness at the centre is t .

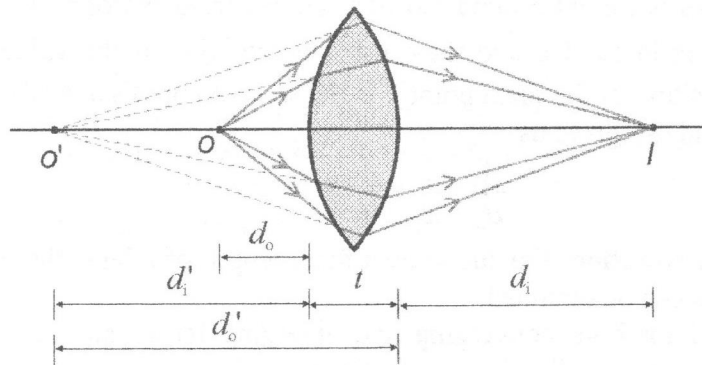


Figure 4-18

We first apply the paraxial equation Eq.(4-6) for the spherical refracting surface to the front surface of the lens and we have

$$\frac{1}{d_o} + \frac{n}{d'_i} = \frac{n-1}{R_1},$$

where R_1 is the radius of curvature of the front surface, $n_1 = 1$ for air and $n_2 = n$ for the lens, d_o is the object distance and d'_i is the image distance for the first surface. Both

distances are measured from the front surface. For the case shown in Fig.4-18, d'_i is negative – the image is virtual at point O' .

Next we apply the paraxial equation to the second surface. The rays striking the second surface seem to come from the point O' . Thus, the object distance for the second surface is $d'_o = -d'_i + t$ (the negative sign is necessary since $d'_i < 0$) and the paraxial equation gives

$$\frac{n}{-d'_i + t} + \frac{1}{d_i} = \frac{1-n}{R_2},$$

where R_2 is the radius of curvature of the second surface (for the case shown in Fig.4-18 R_2 is negative).

When we ignore the thickness of the lens, we set $t=0$, and after adding last two equations we have

$$\frac{1}{d_o} + \frac{1}{d_i} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right). \quad (4-7)$$

This equation relates the object distance d_o to the image distance d_i (the distance from the lens of the final image formed by the lens) and to the properties R_1 , R_2 and n of the lens. It is valid only for paraxial rays and only if the lens is very thin.

If we consider an object at infinity ($d_o \rightarrow \infty$) the image distance equals the focal length $d_i = f$ and from Eq.4-7 we have

$$\frac{1}{f} = (n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right). \quad (4-8)$$

This is called the **lens-maker's equation**. It relates the focal length of any lens to the radii of curvature of its two surface and its index of refraction.

Note that a radius of curvature is positive if a surface is convex to the incoming light and is negative if concave. Or, we can say that $R > 0$ if the centre of curvature is on the side of the lens opposite the light source and $R < 0$ if the centre of curvature is on the same side as the light source.

Note also that if a lens is turned around (so light comes from the opposite direction) R_1 and R_2 exchange roles in Eq.4-8 and they also change sign so the value of f remains the same. Thus the position of the focal point F is the same on both sides of a lens.

Comparing Eqs.4-7 and 4-8 gives us

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}. \quad (4-9)$$

This is called the **lens equation**. For the known focal length of a lens, the image distance for any object distance can be obtained.

This equation is valid for both converging and diverging lenses and for all object and image positions if we take the following conventions:

- the focal length is positive for converging lenses and negative for diverging lenses.
The radius of curvature is positive when light strikes a convex surface and negative when it strikes a concave surface;
- the object distance is positive if it is on the side of the lens from which light is coming;
- the image distance is positive if it is on the opposite side of the lens from where light is coming; if it is on the same side, the image distance is negative;
the image distance is positive for a real image and negative for a virtual image;
- object and image heights are positive for points above the axis and negative for points below the axis;

The **lateral magnification** m of a lens is defined as the ratio of the image height h_i to object height h_o

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \quad (4-10)$$

For an upright image the magnification is positive and for an inverted image the magnification is negative.

Example: What are the position and the size of the image of 22.4 cm high object placed 1.5 m from the convex lens of the focal length 50 mm?

Solution: From the lens equation we can write

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{0.05} - \frac{1}{1.5} = \frac{29}{1.5} \text{ m}^{-1},$$

and so $d_i = \frac{1.5}{29} = 0.052 \text{ m}$. The image is placed 5.2 cm behind the lens. Notice that the image is 2 mm farther from the lens than would be the image for an object at infinity.

The magnification $m = -\frac{d_i}{d_o} = -\frac{0.052}{1.5} = -0.035$,

so $h_i = m \times h_o = (-0.035) \times 22.4 = -0.78 \text{ cm}$ and it is inverted.

Example: Where must a small object be placed if concave lens of the focal length $f = 25 \text{ cm}$ is to form a virtual image at distance 20 cm from the lens?

Solution: Since $f = -25 \text{ cm}$ and $d_i = -20 \text{ cm}$, we can write

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{-25} - \frac{1}{-20} = \frac{1}{100} \text{ cm}^{-1},$$

so $d_o = 100 \text{ cm}$ and thus the object must be placed at point of 100 cm in front of the lens.

4.8 Simple Magnifier

The magnifying glass or the simple magnifier is used to produce magnified images of objects. To examine detail on an object, we must bring it up close to eyes to increase the angle subtended by the object at the eye. The nearest point at which an eye can accommodate is called the **near point**. It varies from person to person. As the standard near point is taken the near point of 25 cm distant. The most distant point that can be focused by eye is called the **far point**. For "normal" eye the far point is at infinity.

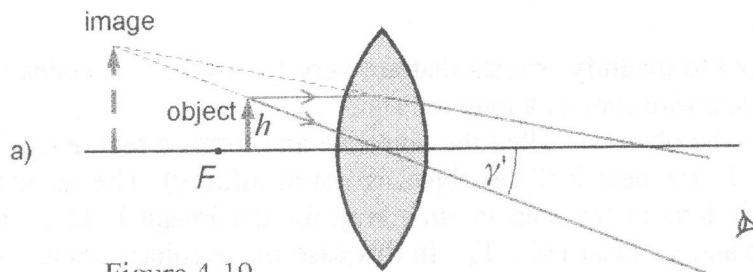
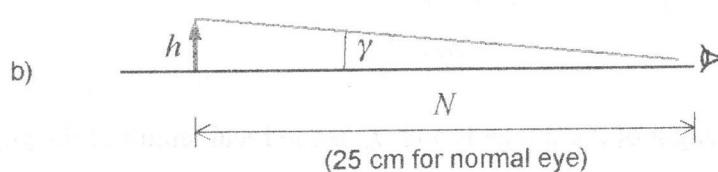


Figure 4-19



In Fig.4-19a we have an object placed at the focal point or at point very close to the focal point. The converging lens produces a virtual and magnified image which must be located between the near point (25 cm) and infinity if the eye is to focus on it.

In Fig.4-19b the same object is viewed at the near point with eye only. It is clear that the angle the object subtends is much larger when the magnifier is used.

The **angular magnification** M of the lens is defined as the ratio of the angle subtended by the object using the lens to the angle subtended with the object at the near point:

$$M = \frac{\gamma'}{\gamma}$$

We now express M in terms of the focal length f of the lens.

First, we will suppose that the image produced by lens is **at the near point** $N=25$ cm (Fig.4-19a). Since $d_i = -N$, applying the lens equation yields for d_o

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{f} + \frac{1}{N},$$

hence

$$d_o = \frac{Nf}{f+N}$$

We see that $d_o < f$; $\left(\frac{d_o}{f} = \frac{N}{f+N} < 1\right)$.

Let h be the height of an object, so

$$\gamma' = \frac{h}{d_o} = \frac{h(f+N)}{Nf} \quad \text{and} \quad \gamma = \frac{h}{N}$$

Thus

$$M = \frac{\gamma'}{\gamma} = 1 + \frac{N}{f} \quad (4-11)$$

If the image is **at infinity**, the object is at the focal point. In this case $\gamma' = \frac{h}{f}$, and

the magnification equals $M = \frac{\gamma'}{\gamma} = \frac{N}{f}$ (4-12)

We see that the magnification is greater when the eye is focused at its near point. We also see that the shorter the focal length of the lens the greater the magnification.

4.9 Telescope

A telescope is used to magnify objects that are very far away. It contains two converging lenses located at opposite ends of a long tube.

The lens closer to the object is called the objective and forms a real image I_1 of the object at its focal point F_o (or near it if the object is not at infinity). The second lens is called the eyepiece. This lens is adjusted in such way for the image I_1 to be placed between the eyepiece lens and its focal point F_e . In this case the eyepiece produces the magnified inverted and virtual image I_2 , seen by the eye.

The **angular magnification** of the telescope is defined as

$$M = \frac{\gamma'}{\gamma}$$

$\gamma \approx \frac{h}{f_o}$, h is the height of the image I_1 and f_o is the focal length of the objective;

$\gamma' = \frac{h}{f_e}$, f_e is the focal length of the eyepiece.

Thus
$$M = \frac{f_o}{f_e} \quad (4-13)$$

It is clear that to obtain a large magnification the objective should have a long focal length and the eyepiece a short one.

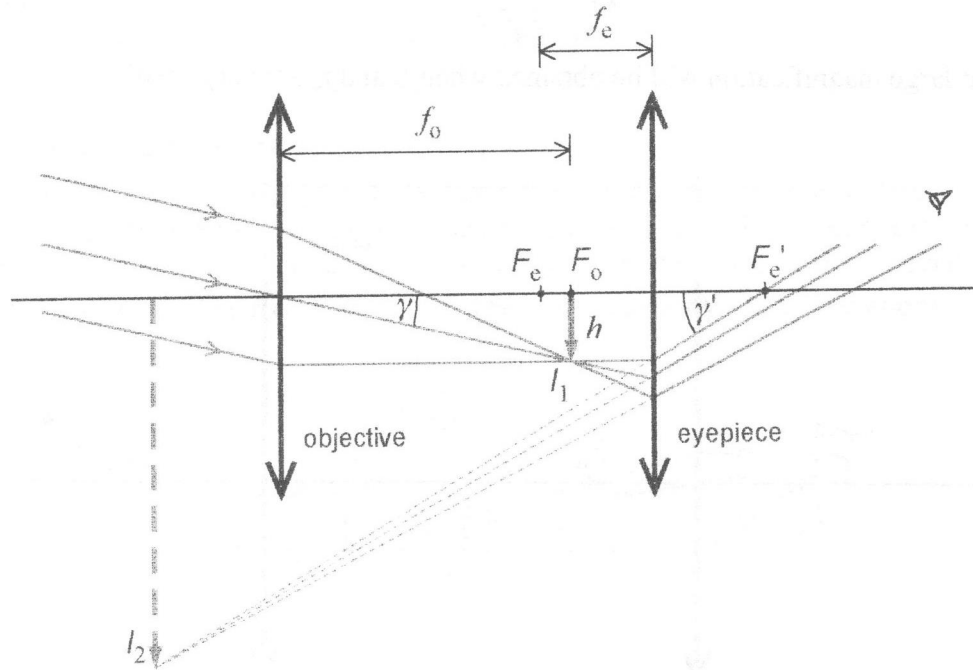


Figure 4-20

4.10 Microscope

A microscope is used to view objects that are very close. It also has both objective and eyepiece.

The object is placed just beyond the objective's focal point F_o . The image I_1 formed by the objective lens is real, quite far from the lens and enlarged and is located close to the eyepiece's focal point F_e between the focal point and the eyepiece lens. This image is magnified by the eyepiece into a very large virtual image I_2 , seen by the eye.

The **total magnification** of the microscope is equal to the product of magnifications produced by both lenses.

According to Eq.4-10 for the magnification of a simple lens we can express the magnification of the image I_1 due to the objective lens:

$$M_o = \frac{d_i}{d_o} \doteq \frac{l - f_e}{d_o}, \quad (4-14)$$

where l is the distance between the lenses (we ignore the minus sign in Eq.4-10 which only tells us that the image is inverted). The eyepiece acts as a simple magnifier and thus from Eq.4-12 its magnification equals

$$M_e = \frac{N}{f_e}, \quad (4-15)$$

where $N=25$ cm is the near point for the normal eye.

Hence, the total magnification of the microscope equals

$$M = M_o \times M_e = \frac{l - f_e}{d_o} \times \frac{N}{f_e}$$

When we consider f_o and f_e small compared to l , then $(l - f_e) \approx l$ and when we put $d_o \approx f_o$, we obtain the formula for this magnification in the practical form

$$M = \frac{lN}{f_e f_o} \quad (4-16)$$

We see that large magnification will be obtained when f_e and f_o are very small.

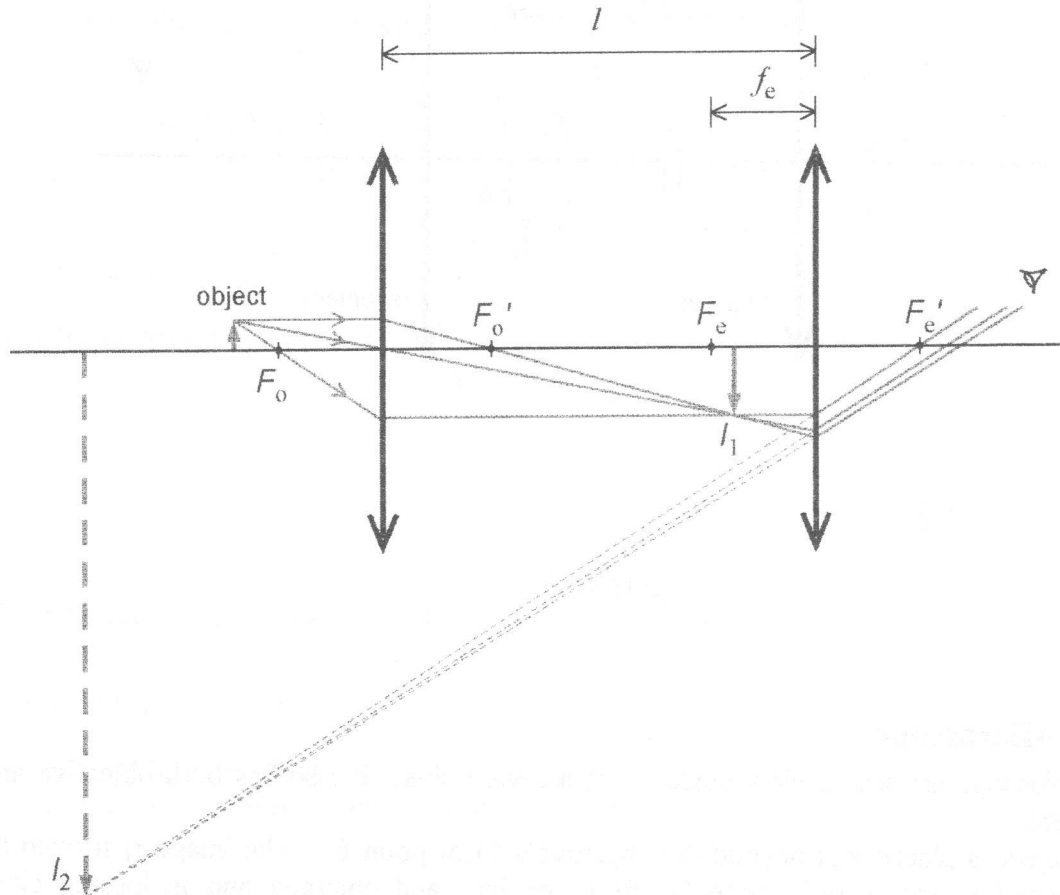


Figure 4-21

Example: A microscope consists of $10\times$ eyepiece and $50\times$ objective 18 cm apart.

Calculate:

- the total magnification,
- the focal length of each lens,
- the position of the object when the final image is focused at infinity.

Solution: $M=10\times 50=500$.

From Eq.4-15 the eyepiece focal length is $f_e = \frac{N}{M_e} = \frac{25}{10} = 2.5$ cm.

To find f_o we first determine d_o by solving Eq.4-14 for d_o

$$d_o = \frac{l - f_e}{M_o} = \frac{18 - 2.5}{50} = \frac{15.5}{50} = 0.31 \text{ cm}.$$

From the lens equation we can find now f_o :

$$\frac{1}{f_o} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{l - f_e} = \frac{1}{0.31} + \frac{1}{15.5} = \frac{51}{15.5} \text{ cm}^{-1}.$$

$$\text{So, } f_o = \frac{15.5}{51} = 0.30 \text{ cm}.$$

We see that the object distance d_o is very close to f_o .

4.11 Lens Aberrations

We have developed a theory of image formation by a thin lens. The obtained results are based on approximation that all rays make small angles with the axis and we can use $\sin \alpha \approx \alpha$ and $\tan \alpha \approx \alpha$. Because of this approximation we expect deviations from the simple theory called lens aberrations. There are several types of aberrations.

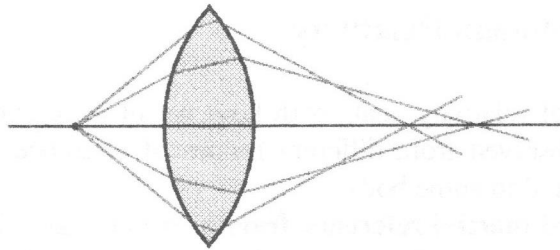


Figure 4-22

If we consider a point object on the axis of a lens then rays that pass through lens close to its centre are focused at a different point than rays passing far from its centre. This is called **spherical aberration**. The image will not be a point but a very small circle surface. Spherical aberration is corrected by the use of several lenses in combination.

For object points off the lens axis, rays passing through different parts of the lens cause spreading of the image that has not shape of the circle. This is called **astigmatism**.

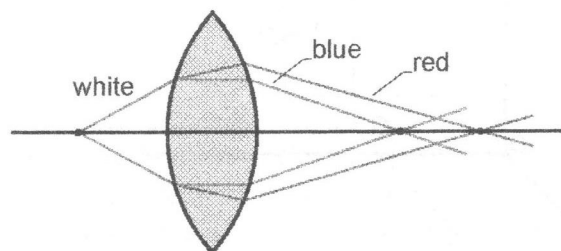


Figure 4-23

If the light is not monochromatic, there will also be **chromatic aberration**. This aberration is due to the variation of index of refraction with wavelength – called dispersion. For example, blue light is bent more than red light by glass. Thus if white light is incident on a lens, the different colours are focused at different points and we observe non sharp coloured image. Chromatic aberration can also be eliminated by the use of two lenses made of different materials with different indices of refraction.