Tečné a normálové zrychlení - odvození

$$(\vec{\boldsymbol{\tau}}^0)^2 = \vec{\boldsymbol{\tau}}^0 . \vec{\boldsymbol{\tau}}^0 = \tau_1^2 + \tau_2^2 + \tau_3^2 = 1$$

$$\vec{\boldsymbol{r}} = \vec{\boldsymbol{r}}(s) = \vec{\boldsymbol{r}}(s(t))$$

$$2\tau_1\tau_1' + 2\tau_2\tau_2' + 2\tau_3\tau_3' = 0$$

$$\vec{\boldsymbol{\tau}}^0 = \vec{\boldsymbol{\tau}}^0(s(t))$$

$$\vec{\boldsymbol{\tau}}' . \vec{\boldsymbol{\tau}}^0 = 0$$

$$\vec{\boldsymbol{a}} = \frac{\mathrm{d}\vec{\boldsymbol{v}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}s}{\mathrm{d}t}\vec{\boldsymbol{\tau}}^0\right) = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}\vec{\boldsymbol{\tau}}^0 + \frac{\mathrm{d}s}{\mathrm{d}t}\frac{\mathrm{d}\vec{\boldsymbol{\tau}}^0}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}s}{\mathrm{d}t}\vec{\boldsymbol{\tau}}^0 + \frac{\mathrm{d}s}{\mathrm{d}t}\frac{\mathrm{d}\vec{\boldsymbol{\tau}}^0}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} =$$

$$v = \sqrt{\vec{\boldsymbol{v}} . \vec{\boldsymbol{v}}} = \sqrt{\left(\frac{\mathrm{d}s}{\mathrm{d}t}\right)^2 \vec{\boldsymbol{\tau}}^0 . \vec{\boldsymbol{\tau}}^0} = \frac{\mathrm{d}s}{\mathrm{d}t}$$

$$x' = \frac{\mathrm{d}x}{\mathrm{d}s}$$

$$= \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\boldsymbol{\tau}}^0 + v^2\vec{\boldsymbol{\tau}}' \qquad = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\boldsymbol{\tau}}^0 + v^2\frac{\vec{\boldsymbol{\nu}}^0}{R}$$

$$\frac{1}{\sqrt{\vec{\boldsymbol{\tau}}' . \vec{\boldsymbol{\tau}}'}} = R$$

$$\vec{\boldsymbol{\tau}}^0 \equiv \frac{\mathrm{d}\vec{\boldsymbol{r}}}{|\mathrm{d}\vec{\boldsymbol{r}}|} = \frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}s} = \vec{\boldsymbol{r}}'$$

$$\vec{\boldsymbol{\nu}}^0 = \frac{\vec{\boldsymbol{\tau}}'}{|\vec{\boldsymbol{\tau}}'|} = \frac{\vec{\boldsymbol{\tau}}'}{\sqrt{\vec{\boldsymbol{\tau}}' . \vec{\boldsymbol{\tau}}'}} = R\vec{\boldsymbol{\tau}}'$$

$$\vec{\boldsymbol{v}} = \frac{\mathrm{d}\vec{\boldsymbol{r}}(s(t))}{\mathrm{d}t} = \frac{\mathrm{d}\vec{\boldsymbol{r}}}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = \vec{\boldsymbol{r}}'\frac{\mathrm{d}s}{\mathrm{d}t} = \vec{\boldsymbol{\tau}}^0\frac{\mathrm{d}s}{\mathrm{d}t}$$

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